

ORAL QUALIFYING EXAMINATION SYLLABUS

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1. MAJOR TOPIC: COMBINATORICS

- (1) Enumeration: counting and double counting, pigeonhole principle, inclusion-exclusion, Stirling's approximation, central limit theorem (Lindeberg-Lévy), binomial identities, Catalan numbers, generating functions
- (2) Set systems: Sperner's theorem, Lubell-Yamamoto-Meshalkin and Bollobás' inequality, Erdős-Ko-Rado theorem, Kruskal-Katona and Harper's theorem, Fisher's inequality and odd/even town, uniform and non-uniform Ray-Chaudhuri Wilson theorem, Frankl-Wilson theorem, Baranyai's theorem
- (3) Posets and lattices
 - Dilworth's theorem (on partition into chains)
 - Birkhoff's representation theorem
 - Correlation inequalities: Harris-Kleitman, Fortuin-Kasteleyn-Ginibre, Ahlswede-Daykin, statement of Sahi's theorem and conjecture, statement of Rinott-Saks $2m$ -function theorem, van den Berg-Kesten inequality, statement of Reimer's theorem
 - Linear extensions: Shepp's theorem $\{x < y\} \uparrow \{x < z\}$; counterexample to $\{w < x < z\} \uparrow \{w < y < z\}$
- (4) Ramsey Theory: Ramsey's theorem for graphs and hypergraphs, infinite Ramsey, the compactness principle, van der Waerden's theorem, Hales-Jewett theorem, Rado's theorem
- (5) Discrepancy theory: Beck-Fiala theorem
- (6) Probabilistic Method
 - Linearity of expectation, union bound and Bonferroni inequalities, Markov's inequality, Chernoff bounds, law of total probability
 - Second moment method: variance and covariance, Chebyshev's inequality, application to thresholds
 - Lovász local lemma
 - Poisson paradigm: basic and extended Janson inequalities
 - Entropy: conditional entropy, fundamental inequality, Shearer's lemma
 - Random graphs: $G_{n,p}$ vs. $G_{n,M}$, Bollobás-Thomason theorem
 - Martingales: definition, the edge-exposure martingale, Azuma's inequality, application to chromatic number
 - Applications: graphs of high girth and high chromatic number, property B, lower bounds on Ramsey numbers, Bregman-Minc inequality
- (7) Graph Theory: Tutte's 1-factor theorem, Brooks's and Vizing's theorems, statement of Galvin's theorem, König's theorem, Hall's marriage theorem,

Menger's theorem, weak perfect graph theorem, Turán's theorem, Erdős-Stone theorem, Szemerédi regularity lemma, graph Laplacians, Cheeger inequality on graphs

- (8) Polytopes: Birkhoff-von Neumann, Edmonds matching and perfect matching polytope theorems

2. MINOR TOPIC: ALGORITHMIC AND ADDITIVE-COMBINATORIAL NUMBER THEORY

(1) Additive Combinatorics

- Minkowski sumsets, exact inverse sum set theorem, doubling constants, Cauchy-Davenport, Ruzsa triangle inequality
- Balog-Szemerédi-Gowers theorem
- Combinatorial Nullstellensatz
- Freiman-Ruzsa, Plünnecke-Ruzsa, Petridis, Ruzsa covering lemma, Freiman over G of bounded exponent
- Freiman homomorphisms, Ruzsa modelling lemma
- Roth's theorem and Behrend's construction
- Sums and Products: Erdős-Szemerédi conjecture, Moreira's Theorem, the $\{x, y, x + y, xy\}$ conjecture

(2) Algorithmic Number Theory

- Hermite normal form, solving linear systems over the integers
- Gram-Schmidt, QR-factorization, lattices, Lenstra-Lenstra-Lovász lattice reduction, application to finding minimal polynomials of algebraic numbers
- Agrawal-Kayal-Saxena primality test
- Berlekamp's algorithm; factoring polynomials over $\mathbb{F}_p$ in randomized $\text{poly}(\text{degree}, \log(p))$ time
- Kraitchik's integer factorization scheme, smooth numbers, quadratic sieve algorithm
- Fast Fourier transform and fast polynomial multiplication