

Oral Qualifying Exam Syllabus

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1. Modular Forms

(a) The Modular Group

- i. The action of $SL(2, \mathbb{Z})$ and congruence subgroups on the upper half-plane
- ii. Fundamental domains
- iii. Cusps, elliptic points, and the compactification of $\Gamma \backslash \mathcal{H}$

(b) Modular Forms for $SL(2, \mathbb{Z})$

- i. Definition of modular and automorphic forms for $SL(2, \mathbb{Z})$
- ii. Holomorphic Eisenstein series
- iii. Ramanujan's discriminant function and the Dedekind eta function
- iv. L -functions for modular forms
 - The trivial estimate for Fourier coefficients of a cusp form
 - Functional equation and analytic continuation

(c) Hecke Operators

- i. The slash and Hecke operators
- ii. Commutativity and self-adjointness of Hecke operators
- iii. Coset representatives for $SL(2, \mathbb{Z}) \backslash M_2(\mathbb{Z})$
- iv. Fourier coefficients and eigenvalues of Hecke eigenforms
- v. Euler product for normalized Hecke eigenforms

(d) Two Converse Theorems

- i. The converse theorem for $f \in S_k(\Gamma(1))$
- ii. Twisted L -functions and their functional equations
- iii. Weil's converse theorem

(e) Rankin-Selberg Method

- i. Nonholomorphic Eisenstein series
- ii. The functional equation and Euler product for $L(s, f \times g)$, for f and g Hecke eigenforms

2. Some Topics from Analytic Number Theory

(a) Dirichlet L -Functions

- i. Euler product
- ii. Theta functions
- iii. Analytic continuation and functional equation

(b) The Γ and ζ Functions

- i. Analytic continuations
- ii. Other basic properties

3. Ergodic Theory

- (a) Measure-Preserving Transformations on Probability Spaces
 - i. Factors and isomorphisms of measure-preserving systems
 - ii. Poincaré Recurrence
 - iii. Ergodicity and equivalent characterizations
 - iv. The associated operator on L^2
 - v. Strong-mixing and weak-mixing
 - vi. Induced transformations and Kakutani skyscrapers
- (b) Ergodic Theorems
 - i. Mean Ergodic Theorem
 - ii. Maximal Ergodic Theorem
 - iii. Birkhoff's Ergodic Theorem
- (c) Continued Fractions
 - i. Recurrence relation and other elementary properties
 - ii. Ergodicity of the Gauss map
 - iii. Badly approximable numbers
- (d) Geodesic Flow on Quotients of the Hyperbolic Plane
 - i. The hyperbolic plane and the action of $PSL_2(\mathbb{R})$
 - ii. The associated geodesic and horocycle flows
 - iii. Closed linear groups and dynamics on their quotients
 - iv. Ergodicity of the geodesic flow