

Oral Exam Syllabus

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1 Major: Probability Theory

1.1 Basics

- Independence
- Borel-Cantelli Lemmas
- Conditional Expectation

1.2 Limit theorems

- Weak and Strong Laws of Large Numbers
- Weak Convergence of Probability Measures
- Characteristic Functions of Random Variables
- Weak Convergence in Terms of Characteristic Functions
- Central Limit Theorem

1.3 Discrete Time Stochastic Processes

- Filtrations
- Adapted and Predictable Processes
- Martingales
- Markov Chains
- Random Walks
- Stopping Times
- Martingale Convergence Theorems

2 Minor: Graph Theory

2.1 Fundamentals

- Basic Graph Definitions
- Bipartite Graphs
- Paths, Trees, Cycles
- Euler's Formula
- Proofs that K_5 and $K_{3,3}$ are non-planar.

2.2 Flows, Connectivity, and Matching

- Konig's Theorem
- Hall's Marriage Theorem
- Tutte's 1-Factor Theorem
- 2-Connected Graphs
- Menger's Theorem

2.3 Probability and Graph Theory

- Adjacency Matrix, Graph Laplacian
- Polya Recurrence
- Gambler's Ruin
- Spectral Gap, Mixing Rate for Random Walk on Regular Graphs
- Conductance and the Cheeger-Buser Inequality

References for Major

- [1] R. Durrett. *Probability: Theory and Examples*. Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, 2010.
- [2] David Williams. *Probability with Martingales*. Cambridge University Press, 1991.

References for Minor

- [3] L. Lovász. *Random Walks on Graph: A Survey*, volume 2. 1993. Available at <https://www.cs.cmu.edu/~15859n/RelatedWork/random-walks-on-graphs.pdf>.
- [4] Russell Lyons and Yuval Peres. *Probability on Trees and Networks*, volume 42 of *Cambridge Series in Statistical and Probabilistic Mathematics*. Cambridge University Press, New York, 2016. Available at <https://rdlyons.pages.iu.edu/>.
- [5] Diestel Reinhard. *Graph Theory*. Springer, Princeton, NJ, 5th edition, 2005.