

Oral Qualifying Exam Syllabus

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Main Topic: Algebraic Geometry

• Schemes

1. **Sheaves:** Definition, presheaves and associated sheaves, morphisms, stalks, pull-back and push-forward.
2. **Schemes:** Affine schemes, Proj, structure sheaf, scheme associated to a variety
3. **First properties of schemes:** Reduced, integral, Noetherian schemes, morphisms of finite type, finite morphisms, closed immersions, dimension, fibre products
4. **Separated and proper morphisms:** Definition of separated and proper morphisms, valuative criteria, projective morphisms, reduced structure of closed subsets, scheme-theoretic image, constructible sets
5. **Coherent and quasi-coherent sheaves:** Definition of $\mathcal{O}_X$ -modules, quasi-coherent and coherent sheaves, constructions on $\mathcal{O}_X$ -modules, invertible sheaves, vector bundles
6. **Divisors:** Weil divisors, Cartier divisors, equivalence for locally factorial schemes, invertible sheaves
7. **Projective morphisms:** Criteria and characterization of projective morphisms, ample and very ample line bundles, blowups
8. **Differentials:** Derivations, module of relative differential forms, sheaves of differentials and nonsingularity, tangent sheaf, canonical sheaf, geometric genus

• Cohomology

1. **Derived functors:** Abelian categories, complexes, derived functors, δ -functors
2. **Cohomology of sheaves:** The category of sheaves of $\mathcal{O}_X$ -modules has enough injectives, some basic vanishing theorems
3. **Cohomology of a Noetherian Affine Scheme:** Vanishing of higher cohomology of quasi-coherent sheaves and Serre's criterion for being affine in terms of vanishing of cohomology

4. **Čech Cohomology:** Definition, isomorphism with regular cohomology for a noetherian separated scheme
5. **Cohomology of projective space:** Calculation using Čech cohomology, Serre vanishing, cohomological criterion of ampleness
6. **Ext Groups and sheaves:** Definition and basic properties
7. **Serre duality:** Dualizing sheaf and duality for a projective scheme, invertibility of dualizing sheaf for local complete intersection, isomorphism between dualizing sheaf and canonical sheaf for nonsingular projective variety

Minor Topic: Symmetric Functions

1. **Partitions:** The notion of partition, partition diagrams, conjugate of a partition, skew diagrams and tableaux, the natural ordering, the hook-length and the content.
2. **The ring of symmetric functions:** The ring Λ of symmetric functions, monomial, elementary, and complete symmetric functions, the generating functions $E(t)$, $H(t)$, and $P(t)$, the involution ω , power sums and the ring of symmetric functions $\Lambda_{\mathbb{Q}}$ over the rationals.
3. **Schur functions:** Definition of Schur polynomial, a determinantal identity, Schur functions.
4. **Orthogonality:** Cauchy formula, an ω -invariant inner product in Λ , orthogonal and orthonormal bases.
5. **Skew Schur functions:** Definition of Skew Schur functions, a determinantal identity, the Kostka numbers, double skew Schur functions.
6. **Transition matrices:** The notion of transition matrix and the transition matrices corresponding to the bases m , e , h , and s .
7. **Relationship to cohomology of Grassmanians:** Schubert cells and Schubert varieties, the homomorphism from Λ to cohomology of the Grassmanian.