

Oral Qualifying Exam Syllabus
Edna Jones

Committee: Alex Kontorovich (chair), Henryk Iwaniec, Stephen Miller, and Jerrold Tunnell

1. Modular Forms

- (a) Dirichlet L-Functions
- (b) The Modular Group
- (c) Modular Forms
 - i. Modular and cusp forms and Fourier expansions
 - ii. The dimensions of $M_k(\Gamma(1))$ and $S_k(\Gamma(1))$
 - iii. Eisenstein Series, the Dedekind η -function, Δ , and the Jacobi triple product formula
 - iv. The Petersson inner product on $S_k(\Gamma(1))$
 - v. The L-functions for modular forms and functional equations
- (d) Hecke Operators
 - i. The slash and Hecke operators on holomorphic functions
 - ii. Coset representatives for $SL(2, \mathbb{Z}) \backslash M_n(\mathbb{Z})$
 - iii. Commutativity and self-adjointness of the Hecke operators
 - iv. Hecke eigenforms and Fourier coefficients
 - v. Euler products for Hecke eigenforms
- (e) The Rankin-Selberg Method
 - i. The nonholomorphic Eisenstein series
 - ii. Analytic continuations and Euler products for the product of two modular forms

2. Ergodic Theory

- (a) Ergodicity, Recurrence and Mixing
 - i. Measure-preserving transformations
 - ii. Poincaré recurrence
 - iii. Ergodicity
 - iv. Unitary operators and their spectral properties
 - v. The mean ergodic theorem
 - vi. Maximal inequality and maximal ergodic theorem
 - vii. Birkhoff's pointwise ergodic theorem
 - viii. Strong mixing
 - ix. Weak mixing and equivalences
- (b) Continued fractions
 - i. The Gauss map and the Gauss measure
 - ii. Badly approximable numbers
- (c) Geodesic flow
 - i. Hyperbolic geometry
 - ii. Geodesic and horocycle flow

References

- [1] Daniel Bump. *Automorphic Forms and Representations*, chapter 1. Cambridge University Press, 1998.
- [2] Manfred Einsiedler and Thomas Ward. *Ergodic Theory with a View Towards Number Theory*. Springer-Verlag, 2011.