Oral Qualifying Exam Syllabus

Scott James

1 Several Complex Variables

- Starting Definitions and Theorems
  - Holomorphicity and Hartog's Separate Analyticity Theorem
  - Basic 1-d theorems (Cauchy Integral, Max Principle, etc)
  - $\bar{\partial}$ equation on $\mathbb{C}^n$
  - Hartog Extension Theorem
  - Bochner Extension Theorem (see final section)
  - Harmonic, Subharmonic, Pluriharmonic, Plurisubharmonic Funtions
- Domains and their Equivalencies
  - Domain of Holomorphy
  - Logarithmically Convex Reinhardt Domains
  - Pseudo-convex and Levi Pseudo-convex
- $\bar{\partial}$ Equation Over Pseudo-Convex Domains
  - Hilbert Space Theory, the Fundamental Inequality
  - The Adjoint, $\bar{\partial}^*$
  - Friderich Smoothing Lemma
  - The Levi Problem
- Complex Analytic Varieties
  - Germs, Weierstrass Preparation and Division Theorem
  - Algebraic Properties of Germs
- Other Things
  - Bergman Kernel/Metric
  - Distributions, Fourier Analysis, Sobolev Space H_s
  - Pseudo-differential Operators
  - C-R Structure

2 Riemannian Geometry

- Basic Manifold Theory
  - Smooth Charts, Smooth Functions
  - Fiber Bundles
  - Differential Forms, Integration
- Geodesics
  - Levi-Civita Connection
  - Parallel Transport and the Geodesic Equation
  - Variations, the First Variation Formula
  - Jacobi Equation, Jacobi Fields
  - The Exponential Map
  - Hopf-Rinow Theorem
- Curvature and Energy
  - Riemannain Curvature Tensor
  - Ricci and Scalar Curvature
  - Second Variaton of Energy
  - Bonnet-Meyers Theorem