

Oral Qualifying Exam Syllabus

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Main Topic: Representation Theory of Finite Groups

1. Group Algebras

- (1) Schur's Lemma
- (2) Wedderburn's Theorem

2. Complex Representations and Characters

- (1) Schur Orthogonality Relations
- (2) Integrality of Characters and Central Characters
- (3) Tensor Products
- (4) Frobenius-Schur Indicator

3. Induction and Restriction

- (1) Frobenius Reciprocity
- (2) Clifford's Theorem
- (3) Clifford Correspondence and Gallagher's Theorem

4. Representations over Subfields of $\mathbb{C}$

- (1) Absolutely Irreducible Representations and Splitting Fields
- (2) Schur Index

5. Projective Representation

- (1) Central Extensions
- (2) Schur Multiplier

(3) Character Triples

Minor Topic: Lie Algebras

1. Basic Concepts of Lie Algebras

(1) Ideals, Homomorphisms, Representations, and Derivations

(2) Solvable and Nilpotent Lie Algebras

(3) Engel's Theorem

2. Semisimple Lie Algebras

(1) Lie's Theorem and Cartan's Criterion

(2) Killing Form

(3) Completely Reducible Representations and Weil's Theorem

(4) Root Systems

(5) Simple Roots and Weyl Group

(6) Classification of (Semi)simple Lie Algebras

3. Representation Theory

(1) Universal Enveloping Algebras

(2) Poincar'e-Birkhoff-Witt Theorem

(3) Serre's Theorem

References

[1] I. M. Isaacs, Character theory of finite groups, Academic Press, New York, 1976.

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[2] J. E. Humphreys, Introduction to Lie algebras and representation theory, Graduate Texts in Mathematics, 9, Springer-Verlag, New York, 1978. MR0499562