

Oral Qualifying Exam Syllabus

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Major Topic: Noncommutative Algebra

Structure theory

- semisimplicity, Wedderburn–Artin theorem, Jacobson radical, Hopkins–Levitzki theorem, Nakayama’s lemma, Goldie’s theorem

Division algebras

- Wedderburn’s little theorem, Frobenius theorem, Cartan–Brauer–Hua theorem, Herstein’s theorem, Brauer group, central simple algebras, Azumaya algebras, quasideterminants

Free groups and algebras

- Ore domains, Hall’s collecting process, Cayley graphs, Nielsen–Schreier theorem, Bergman gap theorem, diamond lemma, Dehn’s algorithm

Lie algebras

Finite-dimensional Lie algebras

- solvable and nilpotent Lie algebras, Engel’s theorem, Lie’s theorem and Cartan’s criterion, Killing form, Weyl’s theorem, representations of $\mathfrak{sl}(2)$, root space decomposition
- root systems, simple roots and Weyl group, Coxeter graphs and Dynkin diagrams, classification theorem
- universal enveloping algebras, Poincaré–Birkhoff–Witt theorem, free Lie algebras, generators and relations, Serre’s theorem

Infinite-dimensional Lie algebras

- generalized Cartan matrices, real and imaginary roots, loop algebras and central extensions, Weyl–Kac character formula, denominator formula, Macdonald identities
- affine Lie algebras, Heisenberg algebras, the Virasoro algebra, twisted and untwisted vertex operators, vertex operator realizations of $\widehat{\mathfrak{sl}}(2)$, vertex operator realizations of $\widehat{A}_n$, $\widehat{D}_n$, $\widehat{E}_n$

Hopf algebras

- bialgebras, Sweedler notation, antipodes, H -modules and H -comodules, Hopf modules, fundamental theorem of coalgebras, coradical filtration, Milnor–Moore theorem

Quantum groups

- $U_q(\mathfrak{sl}(2))$, quantum plane, quantum determinant, $\mathcal{O}_q(\mathrm{SL}(2))$, R -matrices

Minor Topic: Abstract Harmonic Analysis

Lie groups

Compact Lie groups

- orthogonality relations, Peter–Weyl theorem, Weyl character formula, Plancherel formula

Non-compact Lie groups

- unitary dual of $\mathrm{SL}(2, \mathbb{R})$, parabolic induction, principal series, Harish–Chandra’s regularity theorem, asymptotics of matrix coefficients, Kazhdan’s property (T)