

Oral Qualifying Exam Syllabus

Hong Chen

Major Topics: Lie Algebras

1. Basic Concepts
 - (1) Definitions and Examples
 - (2) Solvable, Nilpotent, Simple, and Semisimple Lie Algebras
 - (3) Engel's Theorem
2. Semisimple Lie Algebras and Root Systems
 - (1) Theorems of Lie and Cartan
 - (2) Killing Form, Semisimplicity and Simplicity
 - (3) Weyl's Theorem
 - (4) Representations of $\mathfrak{sl}_2$
 - (5) Root Space Decomposition and Root Systems
 - (6) Simple Roots and Weyl Groups
 - (7) Classification of Irreducible Root Systems
3. Existence Theorem
 - (1) Universal Enveloping Algebra and Poincaré–Birkhoff–Witt Theorem
 - (2) Serre's Theorem
4. Representation of Semisimple Lie Algebras in the BGG Category $\mathcal{O}$
 - (1) Basic Definitions of the Category $\mathcal{O}$
 - (2) Highest Weight Modules, Verma Modules, and Irreducible Finite Dimensional Modules
 - (3) Harish-Chandra's Theorem
 - (4) Characters of Finite Dimensional Modules

Minor Topic: Representations of Finite Groups

1. Basic Concepts
 - (1) Definitions and Examples
 - (2) Irreducible Modules and Schur's Lemma
 - (3) Complete Reducible Modules and Maschke's Theorem
 - (4) Semisimple Algebras and Wedderburn's Theorem
2. Complex Representations and Characters
 - (1) Irreducible Characters
 - (2) Orthogonality Relations
 - (3) Character Table and Group Structure
3. Products of Characters
 - (1) Tensor Products
 - (2) Frobenius–Schur Indicator
4. Induction and Restriction
 - (1) Frobenius Reciprocity
 - (2) Clifford's Theorem
 - (3) Clifford Correspondence and Gallagher's Theorem

References

- [1] J.E. Humphreys. *Representations of Semisimple Lie Algebras in the BGG Category O* , volume 94. American Mathematical Soc., 2008.
- [2] J.E. Humphreys. *Introduction to Lie algebras and representation theory*, volume 9. Springer Science & Business Media, 2012.
- [3] I.M. Isaacs. *Character Theory of Finite Groups*. Academic Press, 1976.