

Oral Qualifying Exam Syllabus for James Holland

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Exam Committee: Eric Allender,
Gregory Cherlin,
Grigor Sargsyan (chair), and
Simon Thomas.

1. Set Theory

i. *Basic Set Theory*

- König's theorem;
- generalized Δ -system lemma;
- Shoenfield's absoluteness theorem;
- the constructible hierarchy, $L \models \Diamond + \text{GCH}$;
- Suslin's problem: $\Diamond$ denies SH, and $\text{MA} + \neg\text{CH}$ implies SH;
- existence of an $\aleph_1$ -Aronszajn tree.

ii. *Advanced Set Theory*

- Forcing theorems, chain conditions, closure conditions;
- forcing CH, $\neg\text{CH}$, and $\Diamond$;
- product forcing;
- iterated forcing, consistency of $\text{ZFC} + \text{MA} + \neg\text{CH}$;
- Cohen forcing;
- Easton forcing;
- measurable cardinals, elementary embeddings, and ultrapowers.

2. Model Theory

- Completeness, compactness, Löwenheim–Skolem, Łoś–Vaught, Tarski–Vaught theorems and tests;
- quantifier elimination;
- omitting types theorem;
- atomic, and prime models;
- saturated models;
- homogeneous models;
- indiscernibles;
- ω -stable theories;
- $\aleph_0$ -categoricity, Morley's categoricity theorem;
- Morley rank, transcendentals.

3. Recursion Theory

- Primitive recursive, and recursive functions;
- recursively enumerable sets;
- the recursion theorem;
- relative computability;
- the arithmetic hierarchy;
- Turing degrees;
- simple, and complete sets.