Oral Exam Syllabus

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1 Combinatorics

Basics counting arguments, generating functions, binomial coefficients, recurrence relations, inclusion-exclusion, Stirling's formula.

Set Systems Sperner's Theorem, LYM inequality, Erdos-Ko-Rado, Dilworth's Theorem, statement of Kruskal-Katona

Lattices geometric and distributive lattices, chains in distributive lattices, linear extensions of posets, the Mobius inversion formula, Weisner's Theorem, Birkhoff representation theorem

Ramsey Theory Ramsey's Theorem for graphs and hypergraphs, infinite Ramsey, Konig's Lemma, upper and lower bounds, probabilistic lower bounds, statements of Hales-Jewett and van der Waerden

2 Graph Theory

Matching Hall's theorem, Konig's theorem, matching algorithms, Augmenting Paths, Tutte's Theorem.

Connectivity and Spanning Trees Menger's Theorem, Max Flow/Min Cut Theorem, Prim's Algorithm, Kruskal's Algorithm, Dijkstra's Algorithm, Matrix Tree Theorem, Cayley's formula, Prufer Codes.

Planarity Euler's Formula, Proof that K_5 and $K_{3,3}$ are not planar, Kuratowski, Wagner's Theorem, Crossing number.

Coloring Chromatic and Edge Chromatic Numbers, List Coloring, Brook's Theorem, Konig's Line Coloring Theorem, Vizing's Theorem, Thomassen's Theorem, 5-color theorem, Galvin's Theorem, perfect graphs: definition and statements of theorems

Extremal Problems Turan's Theorem, Statement of Regularity Lemma and its application to the Erdos-Stone Theorem.

3 Probabilistic Methods

Basics Linearity of Expecation, Markov's Inequality, Chernoff bounds, Chebyshev Inequality, statement of Azuma's Inequality, binomial and Poisson distributions

Second Moment Method thresholds of balanced graphs, clique number, distinct sums Method of alterations: high girth and high chromatic number, R(k,k), independence number, lower bound on property B

Lovasz Local Lemma Symmetric and general versions, application to Ramsey lower bounds.

Poisson Paradigm Janson inequalities, Brun's sieve, threshold for EPIT

4 Computational Complexity

P v. NP Definitions, reducibility, the Cook-Levin Theorem, NP completeness of SAT, independent set, 0/1 integer programming, and directed hamiltonian path, conditions that imply $P \neq NP$

Diagonalization Ladner's Theorem, Oracle Turing Machines and the Baker-Gill-Solovay Theorem Space-bounded complexity: definitions, PSPACE completeness of TQBF, NL completeness of PATH, Savitch's theorem, the Immerman-Szeleplcsenyi Theorem

Separation theorems Time and Space Hierarchy Theorems (deterministic and nondeterministic versions)

Polynomial hierarchy Definitions of Σ_i, Π_i, complete problems, conditions that lead to the collapse of PH.

Circuits $P \subseteq P/poly$, CKT-SAT and alternate proof of Cook-Levin, Characterization of P/poly as TMs with advice, Karp-Lipton Theorem, Meyer's Theorem, existence of hard functions, Nonuniform Hierarchy Theorem, definitions of NC_i, AC_i

Randomization Definitions of RP, BPP and ZPP, error reduction, Sipser-Gacs Theorem, $BPP \subseteq P/poly$, randomized reductions and definition of $BP \cdot NP$

Interactive Proofs definitions, $dIP = NP$, $GNI \subseteq AM$, NP completeness of GI implies $\Sigma_2 = \Pi_2$, $IP = PSPACE$

PCP theorem definitions, equivalence of the 3 versions, hardness of approximation for vertex cover and independent set, $NP \subseteq PCP(poly(n), 1)$