Oral Qualifying Exam Syllabus

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Committee: Alex Kontorovich (chair), Henryk Iwaniec, Stephen Miller, Jerrold Tunnell

1. Automorphic Forms
   (a) The Modular Group
      i. $\mathrm{SL}(2,\mathbb{Z})$ and Congruence Subgroups
      ii. Fundamental Domains for $\mathrm{SL}(2,\mathbb{Z})$ and congruence subgroups
      iii. Cusps and Elliptic Points
      iv. The invariant measure of $\mathcal{H}$ under $\mathrm{SL}(2,\mathbb{Z})$
   (b) Modular Forms
      i. Modular and cusp forms
      ii. Fourier expansions
      iii. The dimensions of $M_k(\Gamma(1))$ and $S_k(\Gamma(1))$
      iv. Eisenstein Series, the Dedekind η function, Δ, and the Jacobi triple product formula
      v. The Petersson inner product on $S_k(\Gamma(1))$
      vi. The L-functions for modular forms and functional equations
   (c) Hecke Operators
      i. The slash and Hecke operators on holomorphic functions
      ii. Coset representatives for $\mathrm{SL}(2,\mathbb{Z})\backslash M_n(\mathbb{Z})$
      iii. Commutativity and self-adjointness of the Hecke operators
      iv. Hecke eigenforms and Fourier coefficients
      v. Euler products for Hecke eigenforms
   (d) The Rankin-Selberg Method
      i. The nonholomorphic Eisenstein series
      ii. Analytic continuations and Euler products for the convolution of two modular forms
2. Analytic Number Theory
   (a) Poisson Summation
   (b) The Mellin transform and the Γ function
   (c) The Riemann ζ function and Dirichlet L-functions
      i. Euler Products
      ii. θ functions
      iii. Analytic continuation and functional equations
   (d) The Prime Number Theorem
3. Homogenous Dynamics in the Hyperbolic Plane
   (a) Geodesic Flow
   (b) Horocycle Flow
   (c) Continued Fractions
   (d) Gauss Maps / Measure