Oral Qualifying Exam Syllabus

Qidong He

1 Equilibrium Statistical Mechanics

1. Overview: microscopic vs. macroscopic descriptions.
2. Energy surface; microcanonical ensemble; ideal gases; Boltzmann's entropy.
3. Alternative equilibrium ensembles: canonical, grand-canonical, etc.; principle of maximum entropy; connections between partition functions and thermodynamics; Boltzmann's principle.
4. Thermodynamic limit: thermodynamic potentials; Gibbs measures; existence; equivalence of ensembles; Legendre transform.
5. Cooperative phenomena: phase diagrams and phase transitions; correlation functions.
6. Ising model: exact solutions; FKG, GKS inequalities; Peierls' argument; Lee-Yang circle theorem.
7. Mean-field theory and long-range potentials: Kac interaction; mean-field limit; LMP model.
8. Pirogov-Sinai theory
   (a) Ising model: low-temperature expansion; contours.
   (b) Elements: ground states; contours; Minlos-Sinai trick; contour model; Peierls' condition.
   (c) Techniques: m-potential; cluster expansion; cutoff weights; isoperimetric inequality.
9. Infinite-volume Gibbs measures
   (a) Gibbs measures: finite-volume vs. infinite-volume; spatial Markov property, DLR equations.
   (b) Gibbsian specifications.
   (c) Existence: quasilocal specifications.
   (d) Uniqueness: sensitivity to boundary conditions, relations to phase transitions; total variation distance, Dobrushin uniqueness theorem.
10. Reflection positivity: torus models; reflections; reflection positive measures; chessboard estimate.

2 Probability Theory

1. Fundamentals: probability space; random variables; distribution, expectation, independence; Markov's inequality; tail σ-algebras, Kolmogorov's 0-1 law.
2. Conditional expectation: definition and properties; Jensen's inequality.
3. Martingales: filtration; adapted process; martingale, supermartingale, submartingale; previsible process; martingale transform; stopping time; Doob's optional stopping theorem; applications to simple random walks.
4. Convergence theorems: Doob's martingale convergence theorem; uniform integrability, Lévy's upward theorem.

References

[1] Sacha Friedli and Yvan Velenik. *Statistical mechanics of lattice systems: a concrete mathematical introduction*. Cambridge University Press, 2017.

[2] Errico Presutti. *Scaling limits in statistical mechanics and microstructures in continuum mechanics*. Springer Science & Business Media, 2008.

[3] David Williams. *Probability with martingales*. Cambridge University Press, 1991.