Oral Qual Syllabus

George Hauser

April 17, 2018

Committee:
Stephen D. Miller (Chair), Alex Kontorovich, Henryk Iwaniec, Jerrold Tunnell.

1 Modular Forms

1. Hyperbolic geometry. Action of $\mathrm{SL}(2,\mathbf{R})$. Invariant measures. Classification of motions. $\mathrm{SL}(2,\mathbf{Z})$ and Congruence Subgroups. Fundamental domains. Cusps and elliptic points.
2. Modular forms and cusp forms. Dimensions of spaces. Examples (Eisenstein series, Δ function, j-function.)
3. Fourier expansion of modular forms and specifically Eisenstein series. Trivial bound on coefficients. L-functions and functional equations.
4. Hecke theory for the full modular group. Hecke operators. Commutativity and Self-adjointness. Hecke eigenforms and their Fourier coefficients. Euler products.

2 Analytic number theory

1. Euler-Maclaurin Summation and Poisson Summation.
2. Primes in arithmetic progression.
3. The prime number theorem.
4. The Riemann zeta function and Dirichlet L-functions. Euler product. Analytic continuation and functional equation. Convexity bound. Zero-free region. Counting zeros.
5. Gauss circle problem.

3 Algebraic number theory

1. Number fields. Rings of integers. Orders. Integral basis and discriminant.
2. Unique factorization and invertibility of ideals. Dedekind domains.
3. Decompositions of primes. Galois extensions. Frobenius element.
4. Minkowski's bound. Class group. Unit theorem.
5. Dedekind zeta function. Class number formula (especially for quadratic fields).

References

[1] Bump, D. *Automorphic Forms and Representations.* (Chapter 1). New York: Cambridge University Press, 1997.

[2] Davenport, H. *Multiplicative Number Theory.* (Chapters 1-15). New York: Springer, 2000.

[3] Iwaniec, H. *Topics in Classical Automorphic Forms.* (Chapter 6). Providence, R.I.: American Mathematical Society, 1997.

[4] Lang, S. *Algebraic Number Theory.* New York: Springer-Verlag, 1994.

[5] Marcus, D.A.. *Number Fields* New York: Springer-Verlag, 1977.