

Oral Qualifying Exam Syllabus

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Major Topic: Conley Index Theory

- **Invariant Sets:** (forward, backward, forward-backward, strongly) invariant set definitions, alpha and omega limit sets, attractor-repeller pair decomposition, Morse decomposition
- **Conley's decomposition theorem:** chain recurrent sets, Conley's decomposition Theorem
- **Order Theory:** Posets, (projective, bounded, distributive) lattice definitions, lattice homomorphisms, Birkhoff's Representation Theorem
- **Lattice Structures in Dynamics:** lattices of invariant sets, attracting and repelling neighborhoods, attractors and repellers; invariance, alpha-, omega-limits as lattice homomorphisms
- **Conley index for flows:** existence of index pairs, Homology Conley index, continuation property
- **Conley index for maps:** filtration pairs, index pairs, Homology Conley index, shift equivalence, continuation property

Minor Topic: Recurrence and Ergodic Theory in Combinatorics

- **(Uniform) Recurrence in Compact Spaces:** Dynamical systems, Automorphisms and Homomorphisms of Dynamical systems, recurrence and uniform recurrence/almost periodicity, minimal systems, symbolic flows, Hilbert's lemma
- **Van Der Waerden's Theorem:** Topological Van der Waerden's Theorem, 1-dimensional Van der Waerden's Theorem, Bowen's Lemma, Multiple Birkhoff Recurrence, Multidimensional Van Der Waerden's Theorem
- **Invariant Measures on Compact Spaces:** Poincaré recurrence, invariant measures for symbolic flows, density and upper density, Szemerédi's Theorem, Poincaré sequences and recurrence
- **Other Combinatorial Results:** piecewise syndeticity, Grünwald's theorem, IP-sets, a refinement of Grünwald's theorem, Hindman's theorem