Syllabus for Oral Qualifying Exam

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1 Partial Differential Equations

1.1 Laplace/Poisson Equation

1. Fundamental Solution
2. Mean Value Formulas and Converse to Mean-Value Property
3. Properties of Harmonic Functions: Maximum principle, Smoothness, Local estimates, Liouville's theorem, Analyticity, Harnack's inequality
4. Green's functions for a Ball and for a Half-Space
5. The Energy Estimates

1.2 Heat Equation

1. Fundamental Solutions
2. Mean value Formula
3. Maximum Principle on Bounded $\overline{U}_T$ and $\mathbb{R}^n \times [0, T]$, the Uniqueness by Maximum
4. Uniqueness and Backward Uniqueness by Energy Method

1.3 Wave Equation

1. Solutions by Spherical Means for Dimension $n = 1, 2$ and 3, d'Alembert's Formula for $n = 1$, Kirchhoff's Formula for $n = 3$, and Poisson's Formula for $n = 2$
2. Uniqueness and Domain of Dependence by Energy Methods

1.4 Sobolev Space

1. Definition of Sobolev Space
2. Approximation by Smooth Functions
3. Extensions and Traces

4. Sobolev Inequalities: Gagliardo-Nirenberg-Sobolev inequality, Morrey's Inequality
5. Compact Embedding Theorem: Rellich-Kondrachov Theorem
6. Poincare's Inequality, Difference Quotients
7. Fourier Transform Method for $H^k(\mathbb{R}^n)$
8. The Space H^{-1}.
9. Spaces involving time

1.5 Second Order Elliptic Equations

1. Definition of Weak Solutions
2. Existence by Lax-Milgram Theorem and Energy Estimates by Fredholm Alternative
3. Interior Regularity
4. Weak and Strong Maximum Principles
5. Harnack's Inequality
6. Eigen values of symmetric elliptic operators

1.6 Second Order Parabolic Equations

1. Definition of Weak Solutions
2. Energy estimates
3. Existence and uniquness of weak solution
4. Improved Regularity
5. Weak and Strong Maximum Principles
6. Harnack's Inequality

1.7 Second Order Hyperbolic Equations

1. Definition of Weak Solutions
2. Energy estimates
3. Existence and uniquness of weak solution
4. Improved Regularity
5. Propogation of diturbances-Finite propogation speed

2 Functional Analysis

2.1 Banach Spaces

1. Metric Spaces, Normed Linear Vector Spaces, Banach Spaces and Hahn-Banach Theorem
2. The Baire Category Theorem, the Uniform Boundedness Principle, the Open Mapping Principle and the Closed Graph Theorem, Banach-Alaoglu Theorem
3. Duality, Reflexivity, Weak Topology and Weak* Topology

2.2 Hilbert Spaces

1. Definition of Hilbert Space and the Dual Space of Hilbert Space
2. Projection Lemma, Riesz Representation Theorem and Lax-Milgram Theorem
3. Orthogonal Sets, Basis, the Bessel's Inequality and the Parseval's Theorem

2.3 Compact Operators

1. Bounded linear Operators
2. Definition, Elementary Properties of Adjoint in Banach Space and Hilbert Space
3. Compact operators in Banach Space
4. Riesz Fredholm Thoerem
5. The Spectrum of a Compact Operators in Banach Spaces
6. The Spectrum of a Compact Adjoint Operators in Hilbert Space
7. Hilbert Schmidt theorem

References

[1] Lawrence C. Evans, *Partial Differential Equations*, AMS, 2010

[2] Haim Brezis, *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, Springer, 2010