

ORAL QUALIFYING EXAM SYLLABUS

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1. SCHEMES

1. **Sheaves:** Definition, presheaves and associated sheaves, morphisms, stalks, pull-back and push-forward
2. **Schemes:** Affine schemes, Proj, structure sheaf, scheme associated to a variety
3. **First properties of schemes:** Reduced, integral, Noetherian schemes, morphisms of finite type, finite morphisms, closed immersions, dimension, fibre products
4. **Separated and proper morphisms:** Definition of separated and proper morphisms, valuative criteria, projective morphisms, reduced structure of closed subsets, scheme-theoretic image, constructible sets
5. **Coherent and quasi-coherent sheaves:** Definition of $\mathcal{O}_X$ -modules, quasi-coherent and coherent sheaves, constructions on $\mathcal{O}_X$ -modules, invertible sheaves, vector bundles
6. **Divisors:** Weil divisors, Cartier divisors, equivalence for locally factorial schemes, invertible sheaves
7. **Projective morphisms:** Criteria and characterization of projective morphisms, ample and very ample line bundles, blowups
8. **Differentials:** Derivations, module of relative differential forms, sheaves of differentials and nonsingularity, tangent sheaf, canonical sheaf, geometric genus

2. COHOMOLOGY

1. **Derived functors:** Abelian categories, complexes, derived functors, δ -functors
2. **Cohomology of sheaves:** The category of sheaves of $\mathcal{O}_X$ -modules has enough injectives, some basic vanishing theorems
3. **Cohomology of a Noetherian Affine Scheme:** vanishing of higher cohomology of quasi-coherent sheaves and Serre's criterion for being affine in terms of vanishing of cohomology
4. **Cech Cohomology:** Definition, isomorphism with regular cohomology for a noetherian separated scheme
5. **Cohomology of projective space:** Calculation using Cech cohomology, Serre vanishing, cohomological criterion of ampleness
6. **Ext Groups and sheaves:** Definition of basic properties

- 7. **Serre duality:** dualizing sheaf and duality for a projective scheme, invertibility of dualizing sheaf for local complete intersection, isomorphism between dualizing sheaf and canonical sheaf for nonsingular projective variety
- 8. **Spectral Sequence:** Filtered complex, double complex
- 9. **Hypercohomology**

3. COMPLEX ALGEBRAIC SURFACES

- 1. **Cohomology:** Riemann-Roch Theorem, Noether's formula, and the genus formula
- 2. **Del Pezzo Surfaces:** Definitions and blow-ups
- 3. **K3 surfaces:** Definition, cohomology, periods, polarizations, examples
- 4. **Enriques surfaces:** Examples and correspondence with K3 surfaces
- 5. **ADE singularities:** quotients, equations and resolutions
- 6. **Enriques-Kodaira classification**
- 7. **Bogomolov-Miyaoka-Yau inequality**

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