

Syllabus for Oral Exam

Han Lu

1 Major Topic: Partial Differential Equation

1.1 Classical solution of laplace equation

- (1) Fundamental solution
- (2) Mean value property
- (3) Maximum principle
- (4) Green's representation formula
- (5) Harnack's inequality
- (6) Liouville theorem
- (7) Perron's method
- (8) Hopf lemma
- (9) Schauder estimate
- (10) Existence results by the method of continuity and fredholm alternative

1.2 Classical solution of heat equation

- (1) Fundamental Solution
- (2) Maximum Principles on bounded $\bar{\Omega}_T$ and $R^n \times [0, T]$
- (3) Backward uniqueness by energy methods.

1.3 Wave equation

- (1) d' Alembert's formula, Kirchhoff's formula.
- (2) Uniqueness and domain of dependence by energy methods.

1.4 Sobolev space

- (1) Definition
- (2) Density
- (3) Extensions and Traces
- (4) Rellich-Kondrachov compact imbedding theorem
- (5) Poincare's inequality and Morrey's inequality
- (6) $W_0^{1,n}$ case

1.5 Weak solution of second order Elliptic equation

- (1) Lax-Milgram theorem and Fredholm alternative
- (2) Maximum principle
- (3) Moser Iteration method and Harnack's inequality
- (4) L^p estimate and Interpolation theorem

1.6 Weak solution of second order Parabolic equation

- (1) Definition of weak solution
- (2) Energy estimates, Galerkin approximation
- (3) Existence and uniqueness.
- (4) Maximum principle.

2 Minor Topic: Riemannian Geometry

- (1) Riemannian metric
- (2) Levi-civita connection
- (3) Curvature tensor, sectional curvature, Ricci curvature, scalar curvature
- (4) Geodesics and exponential map
- (5) Jacobi fields, conjugate points
- (6) First variation and second variation of arc length
- (7) Bonnet-Myers Thm, Hopf-Rinow Thm, Cartan-Hadamard Thm
- (8) Space form
- (9) Hypersurface, second fundamental form
- (10) Differential operator: grad, div and Hess.

References

- [1] Gilbarg, D and Trudinger, N. S, Elliptic Partial Differential Equations of Second Order. Springer 1983
- [2] Evans, L.C. Partial Differential equations. AMS Providence, 1998.
- [3] Shoshichi Kobayashi, Katsumi Nomizu. Foundations of Differential Geometry. Interscience Publishers, 1963.