

Qual Syllabus

Milan Haiman

Committee: Swee-Hong Chan, Jeff Kahn, Konstantin Matveev, Bhargav Narayanan

1 Combinatorics

1. **Enumeration:** Counting and double counting, binomial and multinomial coefficients, pigeonhole principle, inclusion-exclusion, Stirling's formula.
2. **Set Systems:** Sperner's theorem, LYM and Bollobás (set pairs) inequalities, Erdős–Ko–Rado theorem, Kruskal–Katona and Harper's theorem, Fisher's inequality and odd/even town, uniform and nonuniform Ray–Chaudhuri–Wilson theorem, Frankl–Wilson theorem, Baranyai's theorem.
3. **Posets:** Birkhoff representation theorem, Dilworth's theorem, Mirsky's theorem.
4. **Ramsey Theory:** Ramsey's theorem, infinite Ramsey, probabilistic lower bounds, Hales–Jewett, van der Waerden, Erdős–Szekeres.
5. **Matchings:** Hall's matching theorem, Menger's theorem, max flow min cut.
6. **Graph Coloring:** Brooks, Vizing, list-coloring (including Galvin).
7. **Extremal Graph Theory:** Turán's theorem, Erdős–Stone–Simonovits theorem, Kővári–Sós–Turán theorem, Szemerédi's regularity lemma, triangle removal lemma
8. **Probabilistic Methods**
 - **Basics:** Linearity of expectation, law of total probability, Markov, Chebyshev, Chernoff.
 - **Alterations:** Lower bound for diagonal Ramsey numbers, application to graphs with high girth and high chromatic number.
 - **Second Moment Method:** General procedure, application to threshold for containing a fixed subgraph in $G_{n,p}$.
 - **Lovász Local Lemma:** Statement for symmetric and general case, applications to Property B, Ramsey lower bounds, and discrepancy.
 - **Correlation Inequalities:** Harris-FKG inequality, Ahlswede–Daykin theorem, Fishburn–Shepp 'XYZ' inequality.
 - **Martingales:** Bounded differences/ Azuma-Hoeffding inequality, vertex and edge exposure
 - **Janson inequalities:** Statement and applications to chromatic number of $G_{n,1/2}$ and the number of triangles in $G_{n,p}$.
 - **Entropy:** Basic properties, conditional entropy, Shearer's lemma, applications to Bregman–Minc, independent sets in a d -regular graph, triangle-intersecting families, and graph homomorphism inequalities.
9. **Algebraic Methods:** Combinatorial Nullstellensatz, Schwartz-Zippel lemma, Linear algebraic methods

2 Probability Theory

1. **Definitions:** σ -algebras and algebras, measures, measurable functions, random variables, modes of convergence, expectations, independence, characteristic functions
2. **Basic Theorems:** Borel-Cantelli lemma, Kolmogorov's 0 – 1 law
3. **Limit Theorems:** Weak and strong laws of large numbers, central limit theorems
4. **Random walks:** Polya's theorem, electrical networks, harmonic functions
5. **Brownian motion:** Construction/definition, basic properties, distribution of maximum and arcsine laws, Donsker invariance principle
6. **Other topics:** Galton-Watson branching process, Wilson's algorithm for uniform spanning trees