Oral Qualifying Exam Syllabus

Rashmika Goswami
Committee: Michael Saks (Chair), Jeff Kahn, Bhargav Narayanan, Shubhangi Saraf

1 Combinatorics

- **Enumeration**: Bijections, generating functions, binomial and multinomial coefficients, recurrence relations, inclusion-exclusion, Stirling's formula
- **Hypergraphs**: Sperner, LYM inequality, Erdös-Ko-Rado, Kruskal-Katona, Fisher's Inequality, Ray-Chaudhuri Wilson, Frankl-Wilson, Baranyai, Harper, Sauer-Shelah
- **Posets and Lattices**: Dilworth, Birkhoff representation theorem, distributive lattices, linear extensions of posets, Harris-Kleitman, Fortuin-Kasteleyn-Ginibre (FKG), Ahlswede-Daykin, Möbius inversion
- **Discrepancy**: Beck-Fiala, six standard deviations suffice
- **Ramsey Theory**: Ramsey, infinite Ramsey, upper and lower bounds, probabilistic lower bounds, Van der Waerden, Hales-Jewett, Rado's theorem
- **Linear Programming**: Weak duality theorem, strong duality theorem, fractional coverings and matchings
- **Algebraic Methods**: Combinatorial Nullstellensatz, Schwartz-Zippel Lemma, linear algebraic methods

2 Probabilistic Methods

- **Basics**: Linearity of expectation, union bound, Markov's inequality, Bonferroni inequalities, Chernoff bounds, alteration methods
- **Second Moment Method**: Chebyshev's inequality, threshold function for containing a fixed subgraph
- **Local Lemma**: Symmetric and general form, applications to hypergraph discrepancy
- **Poisson Paradigm**: Janson's inequality, isolated vertices in $G_{n,p}$, Brun's sieve
- **Martingales**: Vertex and edge exposures, Azuma's inequality and application to chromatic number, Talagrand's inequality
- **Entropy**: Basic properties, Shearer's lemma, Brégman's theorem

3 Graph Theory

- **Matchings**: König, Hall, Tutte, stable matchings, matching polytopes
- **Connectivity**: Menger's Theorem, Max Flow-Min Cut Theorem, Kruskal's Algorithm, Prim's Algorithm, Dijkstra's Algorithm, Matrix-Tree Theorem, Cayley's Formula, structure of 2-connected graphs
- **Coloring**: 5-color theorem, Brooks, Vizing, Thomassen's 5-list-coloring of planar graphs, perfect graphs, Loász's proof of weak perfect graph theorem, Galvin's proof of Dinitz conjecture
- **Extremal**: Turán's theorem, regularity lemma, counting lemma, Chvatal-Rödl-Szemerédi-Trotter, Erdös-Stone theorem, triangle removal lemma, Roth's theorem

4 Complexity

- **P vs. NP**: Reducibility, Cook-Levin Theorem; NP-completeness of SAT, independent set, 0/1 integer programming, conditions that imply P $\neq$ NP
- **Diagonalization**: Ladner's Theorem, Oracle Turing Machines and the Baker-Gill-Solovay Theorem
- **Space Complexity**: PSPACE completeness of TQBF, NL completeness of PATH, Savitch's Theorem, Immerman-Szelepcsényi Theorem
- **Separation Theorems**: Time and space hierarchy theorems (deterministic and non-deterministic versions)
- **Polynomial Hierarchy**: Σ_i, Π_i, complete problems, conditions that lead to collapse of PH
- **Circuits**: P $\subseteq$ P/poly, Cook-Levin via CKT-SAT, characterization of P/poly as TMs with advice, Karp-Lipton theorem, existence of hard functions, nonuniform hierarchy theorem, definition of AC^i, NC^i
- **Randomization**: RP, BPP, ZPP, error reduction, Sipser-Gács, BPP $\subseteq$ P/poly, randomized reductions, definition of BP $\cdot$ NP
- **Interactive Proofs**: GNI $\in$ AM, NP-completeness of GI implies $\Sigma_2^p = \Pi_2^p$, IP = PSPACE
- **Decision Trees**: Decision tree complexity, certificate complexity, randomized decision tree complexity, sensitivity, block sensitivity, degree, relationships between $s(f), bs(f), C(f), D(f), \deg(f)$, and $R(f)$
- **Circuit Lower Bounds**: Hastad's switching lemma, $\oplus \notin AC^0$, Razborov-Smolensky theorem, sunflower lemma, monotone circuit lower bound for CLIQUE