

Syllabus for Oral Qualifying Exam

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1 Major Topic: Partial Differential Equations

1.1 Laplace equation, heat equation and wave equation

1. Poisson kernel, heat kernel, d'Alembert's and Kirchhoff's formulas
2. Duhamel's principle for non-homogeneous heat and wave equations
3. Mean value properties for Laplace and heat equations
4. Maximum principle and regularity for Laplace and heat equations
5. Energy methods for the three equations

1.2 Classical solution of elliptic equation: Schauder's approach

1. Interior Hölder estimate of Poisson equation, given f in Hölder space
2. Schauder estimate of Poisson equation, given f, u in weighed Hölder spaces
3. Schauder estimate of elliptic equation, given f, u in weighed Hölder spaces
4. The inverse operator of Laplace operator on space $C_{(2-\beta)}^\alpha(B)$
5. The inverse operator of elliptic operator on space $C_{(2-\beta)}^\alpha(B)$: method of continuity
6. Maximum principle for elliptic operator
7. Dirichlet problem of elliptic equation with continuous boundary value on bounded domain

1.3 Sobolev spaces

1. Definition and density property
2. Extension and trace operator
3. Sobolev inequalities
4. Rellich-Kondrachov compactness theorem

1.4 Weak solution of elliptic equations in divergence form

1. Lax-Milgram and Fredholm alternative theorem
2. Existence theorem of weak solution
3. Interior H^2 regularity of weak solution
4. Boundary H^2 regularity of weak solution with zero boundary value

2 Minor Topic: Riemannian Geometry

2.1 Riemannian metric and affine connection

1. Local representation and transformation behavior
2. Covariant derivative and parallel transport
3. Levi-Civita connection

2.2 Curvature

1. Tensorial and symmetric property
2. A characterization of constant sectional curvature
3. Second Bianchi identity and Schur theorem
4. Isometric immersion and second fundamental form
5. Gauss and Codazzi equations

2.3 Geodesic

1. Existence and homogeneity
2. Gauss lemma and local minimizing property
3. Totally normal neighborhood
4. Minimizing geodesic and Hopf-Rinow theorem
5. Divergence theorem on manifold

2.4 Jacobi field

1. First and second variation formula of energy
2. Jacobi field along geodesic
3. Taylor expansion of the length of Jacobi field
4. Bonnet-Myers theorem and Synge theorem
5. Index lemma and Rauch comparison theorem
6. Laplace comparison and Bishop volume comparison theorems

References

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- [2] EVANS, L. C. *Partial differential equations*, 2nd ed. ed. Graduate studies in mathematics ; v. 19. American Mathematical Society, Providence, R.I.
- [3] GILBARG, D. *Elliptic partial differential equations of second order*, 2nd ed., rev. 3rd printing. ed. Classics in mathematics. Springer, New York.