Syllabus of Oral Qualifying exams

Aprameya Girish Hebbar

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1 Major Topic: Ricci Flow

Existence and Properties

- Short time Existence on closed manifolds using De Turck's trick.
- Evolution of geometric quantities along Ricci flow: curvature tensors, lengths, volume, distance.. etc.
- Scalar and tensor maximum principles for parabolic equations and systems.
- Bernstein-Bando-Shi estimates for derivatives of curvature, Doubling time estimates for Ricci flow.
- Blowup of $|\operatorname{Rm}_g|_g$ at the singular time.

Ricci flow on 3-manifolds.

- Positive Ricci curvature at time $t = 0$ is preserved along the flow.
- Convergence of $(T - t)^{-1}g(t)$ to a spherical space form.
- Hamilton - Ivey pinching estimates.

Normalized Ricci flow on 2-manifolds.

- Hamilton's Surface Entropy.
- Derivative and maximum principle estimates for scalar curvature
- Hamilton's Harnack estimates on surfaces.

Ricci and Gradient Ricci Solitons.

- The Soliton equation and properties.
- Quadratic growth of the potential function.

Singularities

- Cheeger-Gromov-type compactness theorem for Ricci flow.
- Singularity types, Point picking, Parabolic rescaling.

Perelman's functionals and no-local collapsing.

- Perelman's $\mathcal{F}, \mathcal{W}$ functionals and their monotonicity along Ricci flow.
- Perelman's no local collapsing theorem and applications.

Harnack estimates

- Hamilton's Trace Harnack estimates, applications.

2 Minor Topic: Elliptic Partial Differential Equations

1. Laplace's equation, Poisson's equation

- The fundamental solution, Green's identities, Mean value property and applications, the maximum principle, C^{∞}-regularity, gradient estimates, Harnack's inequality for Laplace equation.
- Poisson kernel for the unit ball, Harnack's convergence theorem, Lioville property.
- Perron's method and existence of solutions for the Dirichlet problem.

2. Heat equation

- The fundamental solution, the weak and strong maximum principle, gradient estimates, Li - Yau Harnack inequality and classical Harnack inequality for heat equation.
- Energy methods for Heat Equation.

3. Sobolev Spaces

- Definitions, Elementary properties and Approximations.
- Trace and Extensions.
- Sobolev Embedding Theorems.
- Rellich-Kondrachov Compact embedding Theorem, Poincare inequalities.

4. Second order Elliptic Operators in divergence form

- Weak solutions: Lax Milgram Theorem, Fredholm Alternative.
- L^2 Interior Regularity.