Oral Qualifying Exam Syllabus

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I. Conley index theory (major topic)

1. Classical Conley index theory for flows and maps
    1. Isolated invariant sets and isolating neighborhoods
    2. Attractor-repeller pair decompositions: omega- and alpha-limit sets, continuation of pairs, connecting orbits, and characterization by Lyapunov functions
    3. Index pairs and the homological Conley index for flows
    4. The homological Conley index for maps: index maps and shift equivalence
    5. The Ważewski property and continuation of the Conley index
    6. Morse decompositions: flow defined order and coarsening; non-existence of a finest Morse decomposition.
2. Order Theory
    1. Lattices: The connecting lemma, join-irreducible elements, Boolean and projective lattices
    2. Birkhoff's Representation Theorem for finite posets and Birkhoff's Representation Theorem for bounded, distributive lattices
3. Lattices arising from dynamical systems
    1. Lattices of forward and backward invariant sets, attractors, and attracting blocks
    2. Invariance, alpha-limits, and omega-limits as lattice homomorphisms

References

Davey, Brian A., and Hilary A. Priestley. 2002. *Introduction to Lattices and Order*. Second edition. Cambridge: Cambridge University Press.

Kalies, William D., Konstantin Mischaikow, and Robert C. A. M. Vandervorst. 2014. “Lattice Structures for Attractors I.” *Journal of Computational and Applied Mathematics* 1 (2): 307–38.

Kalies, William D., Konstantin Mischaikow, and Robert C. A. M. Vandervorst. 2016. “Lattice Structures for Attractors II.” *Foundations of Computational Mathematics* 16 (5): 1151–91.

Mischaikow, Konstantin, and Marian Mrozek. 2000. "Conley Index Theory." *Handbook of Dynamical Systems II: Towards Applications*, (B. Fiedler, ed.).

II. Probability theory (minor topic)

1. Classical probability theory
    1. The measure theoretic framework of probability theory: Elementary properties of distribution functions, operations on the set of measurable random variables on a probability space, computing expected values with the change of variables formula, the variance and k-th moment of a random variable, the Borel-Cantelli Lemmas
    2. Markov, Chebyshev, and Jensen inequalities
    3. The characteristic function of a random variable; the inversion formula; application to L^2-convergent sequences of Gaussian vectors; application to obtaining Gaussian random variables from known Gaussian random variables
    4. Definition of independent events, random variables, and sigma-fields; pairwise-independence; Dynkin's $\pi - \lambda$ theorem and applications to sufficient conditions for independence; expectation of product of independent random variables; uncorrelated random variables; density of sums of independent random variables;
    5. Convergence in distribution and the Central Limit Theorem
    6. Weak and Strong Laws of Large Numbers
    7. Kolmogorov's 0-1 Law, Kolmogorov's maximal inequality, uniform integrability; applications to zero-one laws of Gaussian processes
2. Gaussian processes
    1. Elementary properties and functions of Gaussian processes (with parameter space being a topological space T): Existence, almost sure boundedness, almost sure convergence, continuity of the canonical (pseudo-) metric on T, metric entropy and log-entropy functions
    2. Gaussian inequalities: The Borell-TIS inequality and corollaries, Slepian's inequality, the Sudakov-Fernique inequality
    3. Orthogonal expansions of Gaussian processes
    4. Gaussian excursion probabilities

References

Adler, Robert J., and Jonathan E. Taylor. 2007. *Random Fields and Geometry*. 1st ed. 2007. New York, NY: Springer New York.

Durrett, Richard. 2005. *Probability : Theory and Examples*. 3rd ed. Belmont, CA: Thomson Brooks/Cole.