Lawrence Frolov

Oral Qual Syllabus

1. Mathematical Physics

a. Newtonian Point Mechanics
   i. Lagrangian formulation: Euler-Lagrange equations, Equivalence to Newton's equation of motion, Noether's Theorem (symmetry and conservation laws).

b. Einstein's Special Relativity
   i. Minkowski Spacetime: Poincaré Group, Lorentz transformations, 4-vectors and invariants (proper time, wave operator etc.).
   ii. The Lorentz Force law for test particles, Maxwell's equations for fields given charges/currents, Derivation of wave equation in Lorentz gauge, Gauge invariance, Covariant form of Maxwell's equations.
   iii. Energy estimates for wave E.Q.

c. General Relativity
   i. Metrics, coordinates and coordinate changes.
   ii. Geodesics, affine connections, Christoffel symbols.
   iii. Curvature, Bianchi Identity.
   iv. Vacuum Solutions to the Einstein Equations and their singularities: Schwarzschild, Reisner Nördstrom, and Ker Newman Black Holes.
   v. Event horizons and Cauchy horizons, maximal extensions, Penrose diagrams, Penrose incompleteness theorem.

d. Quantum Mechanics:
   i. Hamiltonian formulation: Legendre transformations.
   ii. Schrödinger's Equation, The (non-relativistic) hydrogen atom as a 1-body problem, N-body Schrodinger Equation.
   iii. Dirac's Equation.

2. Functional Analysis

a. Banach Spaces
    i. Metric Spaces, Normed Linear Vector Spaces.
    ii. Hahn-Banach Theorem.
    iii. Baire Category Theorem, Uniform Boundedness Principle, Open Mapping Principle, Closed Graph Theorem.
    iv. Duality, Reflexivity, Sequential Compactness, Weak Topology, Weak ∗ Topology, Banach-Alaoglu Theorem, Riesz's lemma.
    v. L_p Spaces, properties, reflexivity, separability, dual, Convolution and Regularization.

b. Hilbert Spaces
    i. Definition of Hilbert Space and its Dual Space.
    ii. Bounded operators, Adjoint operators and properties.
    iii. Projection Theorem.
    iv. Riesz representation theorem.
    v. Orthogonal sets, Bessels inequality.
    vi. Completeness.

c. Fourier Transform on R^n
        1. Properties.
        2. Riemann Lebesgue Lemma.
        3. Fourier Inversion.
        4. Plancherel Formula.

d. Theory of Distributions
    i. Definition of convergence on test functions and continuity of functionals.
    ii. Derivatives and convolutions of distributions and test functions.
    iii. Density of C_c^\infty in D'.

e. Sobolev Spaces
    i. Definition of W^m,p and H_s.
    ii. Equivalent norms.
    iii. Sobolev embedding theorem.

iv. Arzela-Ascoli and Kolmogorov-Riesz-Frechet theorem.
v. Rellich–Kondrachov compactness theorem.

f. Compact Operators
   i. Definition, elementary properties and adjoint.
   ii. The spectrum of a compact and compact self-adjoint operator.
   iii. Fredholm Alternative.
   iv. Spectral invariance and the spectral theorem.

List of Textbooks:

1) **Haim Brezis: Functional Analysis, Sobolev Spaces, and Partial Differential Equations**
2) **Evans: Partial Differential Equations**
3) **Reed and Simon: Functional Analysis**
4) **Misner, Thorne, and Kipp: Gravitation**