ORAL QUALIFYING EXAM SYLLABUS
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Major topic: Combinatorics
Minor topic: Symmetric Functions

1. Combinatorics and Graph Theory

1.1. **Combinatorics.**

- **Enumeration**: Bijections, Generating Functions, Binomial and Multinomial coefficients, Inclusion-Exclusion, Stirling's Formula, Recurrence Relations.
- **Hypergraphs**: Sperner's Theorem, LYM inequality, Erdos-Ko-Rado, Kruskal-Katona, Fisher's Inequality, Baranyai, Ray-Chaudhuri-Wilson, Frankl-Wilson and Harper
- **Partially Ordered Sets**: Dilworth's Theorem, linear extensions of posets, Distributive Lattices, Geometric Lattices, Birkhoff Representation Theorem, Dowling-Wilson, Möbius inversion, Weisner.
- **Correlation Inequalities**: Harris- Kleitman, FKG Inequality, Ahlswede-Daykin, Shepp (XYZ), BK inequality
- **Ramsey Theory**: Ramsey, infinite Ramsey, König tree lemma, probabilistic lower bounds, van der Waerden, statement of Szemerédi, Hales-Jewett theorem, Rado's theorem (Partition Regularity and columns condition), Hindman's theorem

1.2. **Probabilistic Methods.**

- **Basics**: Linearity of expectation, union bound, Bonferroni inequalities, Chebychev's inequality, Markov's inequality, Chernoff bounds, alteration methods.
- **Second Moment Method**: Application to threshold function for containing a fixed subgraph
- **Local Lemma**: Symmetric and general versions, Ramsey lower bounds, hypergraph coloring

- **Poisson Paradigm**: Janson's inequality and the number of triangles in $G_{n,p}$, Brun's sieve and the number of isolated vertices in $G_{n,p}$
- **Martingales**: Vertex and edge exposures, Azuma's inequality and application to chromatic number, Talagrand's inequality
- **Random Graphs**: $G_{n,p}$ versus $G_{n,M}$, monotone properties, Bollobás-Thomason existence of thresholds
- **Entropy**: Basic properties, Shearer's lemma, Brégman's Theorem

1.3. **Graph Theory.**

- **Matchings**: Hall, König, Tutte, doubly stochastic matrices and Birkhoff von Neumann theorem, matching polytopes for bipartite graphs
- **Connectivity**: Kruskal and Prim's spanning tree algorithms, Menger's theorem, max-flow min-cut, Cayley's formula and Prüfer Codes, statement of Matrix-Tree theorem
- **Planarity**: Euler's formula, Kuratowski's theorem, Wagner's theorem
- **Coloring**: 5-color theorem, Brooks, Vizing, perfect graphs, statement of strong perfect graph conjecture and outline of proof idea, existence of high girth high chromatic number graphs, list coloring, Galvin's theorem
- **Extremal**: Turan, regularity lemma, counting lemma, triangle removal lemma

2. Partitions and Symmetric Functions

2.1. **Partitions.**

- **Definitions**: Definition of partitions, Ferrers diagrams, conjugates of partitions, skew diagrams
- **Tableaux**: Young tableaux, skew tableaux, Hook length formula

2.2. **Symmetric Functions.**

- **Algebra**: definition of associative algebra of symmetric functions, specializations, the e/h/p/m symmetric functions, the E/H/P generating functions and associated identities, involution ω, inner product
- **Schur functions**: Definition of Schur functions, Kostka numbers, Littlewood Richardson rule
- **Bijections**: RSK, Cauchy Identity (and skew version), Jacobi Trudi, Gessel-Viennot