

Oral Qualifying Exam Syllabus

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1 MAJOR TOPIC: FINITE-DIMENSIONAL LIE ALGEBRAS

1.1 Elementary Notions and Basic Theory

- Definitions, examples, representations, modules.
- Solvable, nilpotent, simple, semisimple Lie algebras and the Killing form.
- Theorems of Lie and Engel.
- Cartan's criteria for semisimplicity and solvability.
- Semisimple Lie algebras as direct products of simple Lie algebras.
- Weyl's theorem for complete reducibility of modules for semisimple Lie algebras.

1.2 Semisimple Lie Algebras

- Representations of $\mathfrak{sl}(2, \mathbb{C})$
- Root systems and axiomatics.
- Simple roots and the Weyl group.
- Construction of root systems and automorphisms.

1.3 Representation Theory

- Universal Enveloping Algebras.
- PBW theorem.
- Free Lie Algebras.
- Generators and relations, Serre's theorem.
- Finite-dimensional modules of semisimple Lie algebras.

2 MINOR TOPIC: TENSOR CATEGORIES

2.1 Definitions and Properties

- (Rigid) Monoidal categories and monoidal functors.
- Mac Lane strictness theorem.
- Tensor categories and Braiding.

2.2 Examples of Tensor Categories

- Category of finite-dimensional representations of a semisimple Lie algebra $\mathfrak{g}$.
- Category of finite-dimensional representations of $U_q(\mathfrak{sl}_2)$.

References

- [1] P. Etingof, S. Gelaki, D. Nikshych and V. Ostrik, Tensor Categories, Math. Surveys Monogr. 205, Amer. Math. Soc., Providence, RI, 2015.
- [2] J.E. Humphreys, Introduction to Lie Algebras and Representation Theory, second printing, revised, Graduate Texts in Mathematics, vol. 9, Springer-Verlag, New York, 1978.
- [3] J. C. Jantzen, Lectures on Quantum Groups, Grad. Stud. Math. 6, Amer. Math. Soc., Providence, RI, 1996.