Oral Qualifying Exam Syllabus

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1 Combinatorics

Fundamentals

- **Enumeration**: Counting and double counting, binomial and multinomial coefficients, pigeonhole principle, Inclusion-Exclusion, Stirling's formula.
- **Set Systems**: Sperner's theorem, LYM and Bollobás (set pairs) inequalities, Erdős–Ko–Rado theorem, Kruskal–Katona and Harper's theorem, Fisher's inequality and odd/even town, uniform and nonuniform Ray-Chaudhuri–Wilson theorem, Frankl–Wilson theorem, Baranyai's theorem.
- **Posets**: Birkhoff representation theorem, Dilworth's theorem.
- **Ramsey Theory**: Ramsey's theorem, infinite Ramsey, compactness principle, probabilistic lower bounds, Hales–Jewett, van der Waerden, Erdős–Szekeres.

Graph Theory

- **Matchings**: König's theorem, Hall's matching theorem, Tutte's 1-factor theorem.
- **Connectivity**: Kruskal and Prim spanning tree algorithms, Menger's theorem, max-flow min-cut.
- **Planarity**: Euler's formula, Kuratowski's theorem.
- **Colouring**: Brooks' theorem, Vizing's theorem, Thomassen's theorem on list-colouring of planar graphs, Galvin's theorem, statements of the weak perfect graph theorem and the strong perfect graph theorem.
- **Extremal**: Mantel's theorem and Turán's theorem, Erdős–Stone–Simonovits theorem, Kővarí–Sós–Turán theorem, statement of Szemerédi's regularity lemma.

Probabilistic Methods

- **Basics**: Linearity of expectation, law of total probability, Markov, Chebychev, Chernoff.
- **Alterations**: Lower bound for diagonal Ramsey numbers, application to graphs with high girth and high chromatic number.
- **Second Moment Method**: General procedure, application to threshold for containing a fixed subgraph in $G_{n,p}$.
- **Lovász Local Lemma**: Statement for symmetric and general case, applications to Property B, Ramsey lower bounds, and discrepancy.
- **Discrepancy**: Beck–Fiala theorem.
- **Correlation Inequalities**: Harris' inequality, Kleitman's lemma, Ahlswede–Daykin theorem, FKG inequality, Fishburn–Shepp 'XYZ' inequality.
- **Martingales and Tight Concentration**: Azuma's inequality, vertex and edge exposure, application to chromatic number of $G_{n,1/2}$.
- **The Poisson Paradigm**: Brun's sieve, the basic and extended Janson inequalities, application to the number of triangles or isolated vertices in $G_{n,p}$.
- **Entropy**: Basic properties, conditional entropy, Shearer's lemma, application to Bregman–Minc permanent inequality, application to counting independent sets in a d-regular graph, application to upper bound on the size of a triangle-intersecting family of graphs.

2 Probability Theory

- **Fundamentals**: Measure-theoretic foundation, probability spaces, random variables, distributions, expectation and variance, independence, Borel–Cantelli lemmas, tail σ-algebra and Kolmogorov's 0-1 law.
- **Convergence of Random Variables**: Types of convergence (almost-sure, in probability, in L^p), weak and strong laws of large numbers, Kolmogorov's three series theorem, uniform integrability.
- **Conditional Expectation**: Definition and basic properties.
- **Discrete-time Stochastic Processes**: Filtrations, adapted and predictable processes, martingales, stopping time, Doob's optional sampling theorem, Upcrossing lemma, Supermartingale convergence theorem, Doob decomposition.