

# ORAL QUALIFYING EXAM SYLLABUS

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## 1. MAJOR TOPIC: LIE ALGEBRAS

### 1.1. Elementary notions and basic theory.

- Definitions, examples, representations, modules
- Solvable, nilpotent, simple, semisimple Lie algebras and the Killing form
- Theorems of Lie and Engel
- Cartan's criteria for semisimplicity and solvability
- Semisimple Lie algebras as direct products of simple Lie algebras
- Complete reducibility of modules for semisimple Lie algebras

### 1.2. Semisimple Lie algebras.

- Representations of  $\mathfrak{sl}(2, \mathbb{C})$
- Root systems and axiomatics
- Simple roots and the Weyl group
- Construction of root systems and automorphisms

### 1.3. Representation theory.

- Universal enveloping algebras
- Poincaré-Birkhoff-Witt theorem
- Free Lie algebras
- Generators and relations, Serre's theorem
- Finite-dimensional modules of semisimple Lie algebras

## 2. MINOR TOPIC: VERTEX OPERATOR ALGEBRAS

### 2.1. Definitions and properties.

- Formal calculus
- Definition and basic properties of vertex operator algebras
- Weak commutativity and associativity of vertex operator algebras
- Definition and basic properties of modules

### 2.2. Examples of vertex operator algebras.

- Vertex operator algebras and modules associated to the Virasoro algebra
- Vertex operator algebras and modules associated to affine Lie algebras
- Vertex operator algebras and modules associated to Heisenberg algebras

## MAIN REFERENCES

- [1] J.E. Humphreys. *Introduction to Lie Algebras and Representation Theory*. Springer, New York, 1972.
- [2] J. Lepowsky, H. Li. *Introduction to vertex operator algebras and their representations*. Birkhäuser, Boston, 2004.