Oral Qualifying Exam Syllabus

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1 Major Topic: Stochastic Calculus

1.1 Stochastic Processes

- Filtration
- Stopping time, Optional time
- Martingale, Submartingale, Supermartingale, Local martingale
- Upcrossing inequality
- Submartingale convergence theorem
- Doob's maximal inequality
- Optional sampling theorem
- Doob-Meyer decomposition
- Square-integrable martingales
- Quadratic variation

1.2 Brownian Motion

- Brownian motion, Construction of Brownian motion
- Markov property
- Reflection principle
- Distribution of first passage time
- Strong law of large numbers for standard Brownian Motion
- Levy's characterization theorem

1.3 Stochastic Integration

- Construction of stochastic integral
- Ito's rule
- Martingale representation theorem
- Burkholder-Davis-Gundy inequalities
- Girsanov theorem
- Novikov condition

1.4 Stochastic Differential Equations

- Strong solutions, Existence and uniqueness
- Weak solutions, Existence and uniqueness
- Linear stochastic differential equations
- Ornstein-Uhlenbeck process

2 Minor Topic: Mathematical Finance

2.1 Risk-Neutral Pricing

- Risk-neutral measure
- Fundamental theorems of asset pricing

2.2 Connections with PDE

- Feynman-Kac theorem
- Black-Scholes-Merton equation

2.3 Black-Scholes Model

- Construction of self-financing strategies
- Absence of arbitrage

2.4 Arbitrage-free Pricing in Black-Scholes Model

- Black-Scholes PDE for standard European options
- Risk-neutral valuation for European options
- Valuation of American options

2.5 Optimal investment in Black-Scholes model

- Hamilton-Jacobi-Bellman equation
- Martingale characterization of optimal investment strategy