

Oral Exam Syllabus

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I Experimental Math

Basics

Dynamic programming, Linear programming, Duality, Identities (Trig, Symmetric function, Elliptic), Proof machines, Proof by differentiation.

Hypergeometric Algorithms

Hypergeometric series: definition and canonical form. β and Γ functions. Similar hypergeometric terms.

Identities: Gauss, Kummer, Saalschütz, Dixon, Clausen, Dougall

Algorithms: Sister Celine, Gosper, Zeilberger, The WZ Phenomenon, Hyper Algorithm

The Fundamental Theorem, Generalizations to q -analogs.

Reference: Ptekovsek, Wilf, and Zeilberger, $A = B$.

Recurrences

Linear: c -Finite, p -Finite, (Non)-Homogenous, Telescoping, Solving.

Non-linear: Somos-sequence, Catalan Numbers.

Maple

Writing psuedo-code. Recursion, Iteration. Greedy algorithm. Ansatz.

II Combinatorics and Graph Theory

II.1 Combinatorics

Enumeration

Basics: Partitioning, Binomial/Multinomial Coefficients, Stirling Numbers, Inclusion/Exclusion, Bijections

Generating Functions: Ordinary, exponential, Polyá enumeration, systems of equations, Lagrange inversion.

Sum-sets

Sum-Product Theorem, Size bounds, Additive Energy, Nullstellensatz.

Hypergraphs

Discrepancy, LYM inequality, Sperner's Theorem, Erdős-Ko-Rado, Van der Waerden's Theorem, Schur's Theorem, Fisher's Inequality, Statement of Kruskal-Katona.

Probabilistic Method

Linearity of Expectation, Markov's Inequality, Chebyshev's inequality, Chernoff bounds, Alteration method, Binomial/Poisson Distributions, Lovasz Local Lemma

II.2 Graph Theory

Basics

Bipartite, Trees, Paths/Cycles/Walks/Circuits, Hamiltonian, Hall's Theorem, König's Tree Lemma.

Planarity

Euler's formula, Kuratowski's Theorem, Wagner's Theorem, Proofs that K_5 and $K_{3,3}$ are nonplanar, Crossing number, Szemerédi-Trotter Incidence Theorem.

Connectivity

k-Connected Graphs, Menger's Theorem, Max Flow/Min Cut Theorem, Prim's Algorithm, Kruskal's Algorithm, Dijkstra's Algorithm, Bellman-Ford Algorithm.

Colorings

Five-Color Theorem, Brook's Theorem, Vizing's Theorem, Perfect Graph Theorem.

Extremal

Ramsey's Theorem, Bounds, Infinite Ramsey, Turán's Theorem. Erdős-Stone Lemma, Statement of Regularity Lemma.