

Oral qualifying exam syllabus

Quentin Dubroff

Committee: Eric Carlen, Jeff Kahn, Bhargav Narayanan, Shubhangi Saraf

1 Primary topic: Combinatorics

1.1 Combinatorics

- **Enumeration:** Bijections, binomial and multinomial coefficients, inclusion-exclusion, Bonferroni inequalities, Stirling's approximation, Prüfer codes, generatingfunctionology
- **Set systems:** Sperner's theorem, Fisher's inequality, Uniform and non-uniform Ray-Chaudhuri–Wilson, Frankl–Wilson, Disproof of Borsuk's conjecture, Erdős–Ko–Rado, Kruskal–Katona, vertex and edge isoperimetry in the cube, VC dimension, Baranyai's theorem
- **Posets:** Dilworth, distributive lattices, Birkhoff representation theorem
- **Ramsey theory** Ramsey's theorem, infinite Ramsey, probabilistic lower bounds, Hales–Jewett, van der Waerden, Erdős–Szekeres

1.2 Graph Theory

- **Connectivity:** Menger's theorem, max-flow-min-cut
- **Matchings:** Hall's theorem, König, Tutte's one-factor theorem, Birkhoff–von Neumann
- **Planarity:** Euler's formula, Kuratowski's theorem, the crossing lemma, five-color theorem
- **Coloring:** Existence of graphs with high girth and high chromatic number, Brooks' theorem, Vizing's theorem, Thomassen's five-list-coloring of planar graphs, Galvin's proof of the Dinitz conjecture, Lovász's proof of the weak perfect graph theorem, statement of strong perfect graph theorem
- **Extremal:** Turán's theorem, statement of Szemerédi's regularity lemma, Erdős–Stone, counting lemma, triangle removal

1.3 Probabilistic Methods

- **Basics:** Linearity of expectation, law of total probability, alterations, Markov, Chebychev, Chernoff
- **Second Moment Method:** General procedure, application to threshold for containing a fixed subgraph in $G_{n,p}$
- **Lovász Local Lemma:** Statement for symmetric and general case, application to Ramsey lower bounds, application to discrepancy
- **Correlation Inequalities:** Harris, Kleitman, Ahlswede-Daykin, FKG
- **Martingales and Tight Concentration:** Azuma's inequality, vertex and edge exposure, application to chromatic number of $G_{n,1/2}$, Talagrand's inequality
- **The Poisson Paradigm:** Brun's sieve, the Janson inequalities, application to the number of triangles or isolated vertices in $G_{n,p}$
- **Entropy:** Basic properties, Shearer's lemma, application to Minc conjecture

2 Convex and discrete geometry

- **Basics:** Jensen's inequality, support properties, separation
- **Combinatorial geometry:** Helly's theorem, Radon's theorem, Caratheodory's theorem, the Szemerédi-Trotter theorem
- **Steiner Symmetrization:** The sphericity theorem, monotonicity of key properties, application to isoperimetric, isodiametric, Blaschke-Santaló and Brunn-Minkowski inequalities
- **Brunn-Minkowski Theory:** Prékopa-Leindler inequality, extension of Brunn-Minkowski inequality to non-convex bodies, concentration of measure in Gaussian space, statement of concentration of measure on the sphere, Dvoretzky's theorem
- **Miscellaneous** John's ellipsoid theorem