Oral Qualifying Exam Syllabus

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1. **Major Topic: Elliptic PDE**
   (a) Properties of Harmonic Functions
      i. Mean value property
      ii. Maximum principles
      iii. Harnack inequality, Liouville Theorem
      iv. Smoothness and analyticity
      v. Green's function, Poisson formula
      vi. Removable Singularities
      vii. Weyl's Lemma
   (b) Sobolev Spaces
      i. Approximation by smooth functions
      ii. Extension and trace operators
      iii. Sobolev Inequalities: Gagliardo-Nirenburg-Sobolev, Morrey, Poincare
      iv. Rellich-Kondrachov Compactness Theorem
      v. The space $H^{1/2}$
   (c) Weak Solutions to Divergence Form Equations
      i. Lax-Milgram Theorem
      ii. Fredholm Alternative
      iii. Existence theorems for weak solutions
      iv. Compact and adjoint operators
      v. Spectrum and Eigenvalue Problems
   (d) Maximum Principles for Elliptic Equations
      i. Weak maximum principle
      ii. Hopf's lemma and strong maximum principle
      iii. Harnack's inequality

2. **Minor topic: Functional Analysis and Fourier Analysis**

   (a) Banach Spaces
      i. Hahn-Banach Theorem
      ii. Weak and weak*-topology
      iii. Banach-Alaoglu theorem
      iv. Riesz's lemma

   (b) Hilbert Spaces
      i. Cauchy-Schwarz and parallelogram law
      ii. Riesz Representation theorem
      iii. Orthonormal bases
      iv. Bessel's inequality

   (c) Fourier Analysis
      i. Fourier transform on $\mathbb{R}^n$
      ii. Derivatives of Fourier transform
      iii. Plancherel theorem
      iv. Properties of convolutions
      v. Hausdorff-Young inequality