

Oral Qualifying Exam Syllabus

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1 First Topic: Central Simple Algebras and Galois Cohomology

1. Quaternion Algebras (Gille and Szamuely Ch.1)
 - Basic properties
 - Splitting over a quadratic extension
 - The associated conic and Witt's theorem
 - Tensor products of quaternion algebras and Albert's theorem
2. Central Simple Algebras and Galois Descent (Gille and Szamuely Ch.2.1-2.7, Milne p.136-140)
 - Wedderburn's theorem
 - Splitting fields
 - Galois descent
 - The Brauer group
 - Reduced norms and traces
 - Azumaya Algebras
3. Techniques from group cohomology (Gille and Szamuely Ch.3.1-3.3)
 - Definition of cohomology groups
 - Explicit resolutions
 - Relation to subgroups
4. The Cohomological Brauer group (Gille and Szamuely Ch.4.1-4.5, 4.7)
 - Profinite groups and Galois groups
 - Cohomology of profinite groups
 - Cohomological interpretation of the Brauer group
 - Period and index
 - Cyclic algebras
5. Cohomology of Local Fields (Gille and Szamuely Ch.6.3-6.5)
 - Cohomology of Laurent series fields
 - Cohomology of function fields of curves
 - Applications to class field theory
6. Separable algebras (Milne p.20-23)

2 Second Topic: Commutative Algebra

1. Basic Constructions (Atiyah and Macdonald Ch.1-11, Milne p.32-36)

- Basic properties of rings and modules
- Rings and modules of fractions
- Primary decomposition
- Integral dependence and valuations
- Artinian and Noetherian rings
- DVR's and Dedekind domains
- Completions and Henselian rings
- Dimension theory

References

- [1] Philippe Gille and Tamás Szamuely. *Central Simple Algebras and Galois Cohomology*. [Cambridge Studies in Advanced Mathematics, vol. 101]. Cambridge University Press, 2006.
- [2] James S. Milne. *Étale Cohomology*. [Princeton Mathematical Series, vol. 33]. Princeton University Press, 1980.
- [3] M.F. Atiyah and I.G. Macdonald. *Introduction to Commutative Algebra*. Addison-Wesley, 1969.