

Oral Exam Syllabus
Madison Crim

Major Topic: Lie Algebras

(1) Basic Concepts

1. Definitions and Examples
2. Solvable, Nilpotent, Simple, and Semisimple Lie Algebras
3. Engle's Theorem

(2) Semisimple Lie Algebras and Root systems

4. Lie's Theorem and Cartan's Criterion
5. Killing Form
6. Weyl's Theorem
7. Representation of $\mathfrak{sl}(2, F)$
8. Root Space Decomposition and Root systems
9. Simple Roots and Weyl Groups
10. Classification of Irreducible Root Systems

(3) Representation Theory of Semisimple Lie Algebras

11. Universal Enveloping Algebra
12. Universal Enveloping Algebra center
13. Poincare-Birkhoff-Witt Theorem
14. Serre's Theorem
15. Highest Weight Modules, Verma Modules, and Irreducible Finite Dimensional Modules

Minor Topic: Algebraic Number Theory

1. Number fields
2. Rings of integers
3. Integral bases
4. Discriminant
5. Decompositions of primes
6. Minkowski's bound
7. The class group
8. The unit theorem
9. Dedekind zeta function
10. Class number formula