SYLLABUS FOR ORAL QUALIFYING EXAM

BAOZHI CHU

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MAJOR TOPIC: Partial Differential Equations

- Laplacian operator and Poisson Equations

  (1) Mean value property, Maximum principles, Average Monotonicity
  (2) Fundamental solution and Green's representation
  (3) Green's function and Poisson's Formula, Smoothness, Analyticity, Convergence theorem, Harnack inequality, Liouville Theorem, removable singularity
  (4) Potential Theory for Poisson equation: Regularity estimate and Dirichlet Problems
  (5) The Calderon-Zygmund estimate for Laplacian

- Classical non-divergence form second order linear elliptic equations

  (1) Weak Maximum Principle, Hopf's Lemma and Strong Maximum Principle, A Priori Estimate
  (2) Interior Schauder Estimate, Pertubation via. Freezing coefficients
  (3) Interior solvability of Dirichlet BVP, Method of Continuity, Perron's Method
  (4) Boundary Schauder Estimate: Straightening Boundary Method

- Sobolev Spaces

  (1) Definitions and Elementary properties
  (2) Approximations, Chain Rule
  (3) Traces and Green's formula
  (4) Extensions
  (5) Sobolev Embedding Theorems, Poincare inequalities
  (6) Compactness Theorems: Rellich-Kondrachov Theorem and Sobolev Compactness via. Banach- Alaoglu Theorem

- Weak solutions for Divergence form second order linear elliptic equations

  (1) The Lax-Milgram Lemma and L^2 existence
  (2) A Schauder Theorem for Holder estimate of the first order derivative: Method of Companato; Completeness of Companato's Holder pseudo-norm
  (3) Moser's Harnack inequality: Cacciopolli's inequality, Local boundedness of solutions via. Moser's iteration, BMO spaces, John-Nirenberg inequality and its Corollaries, Proof of Moser's Harnack inequality
  (4) Applications of Moser's Harnack inequality: De Giorgi-Nash-Moser regularity, Liouville

Theorem, Strong Maximum Principle for generalized solutions

MINOR TOPIC: Riemannian Geometry

- Connections

(1) Definitions, Connections in the tangent bundle, Christoffel Symbols
(2) Covariant derivatives of tensor fields
(3) Vector and tensor fields along curves
(4) Existence and uniqueness of geodesics and parallel transports

- Riemannian Metric and The Levi-Civita Connection

(1) Definition of Riemannian metric and local representations
(2) Raising and lowering indices, Trace operator, Inner product and norm of tensors
(3) Riemannian volume form
(4) Lengths and distances
(5) Metric connection, Symmetric connection, Levi-Civita connection
(6) The exponential Map
(7) Normal neighborhood and normal coordinates
(8) Divergence, Laplacian, Divergence theorem and Green's identities

- Geodesics

(1) Symmetry Lemma, First Variation Formula
(2) The Gauss Lemma
(3) Uniformly Normal coordinates
(4) Minimizing properties
(5) Completeness, Hopf-Rinow

- Curvature and Riemannian Submanifolds

(1) The curvature tensor and its symmetries
(2) Ricci and Scalar curvatures
(3) The second fundamental form, The Gauss formula, The Weingarten equation, The Gauss equation, The Codazzi equation
(4) Hypersurfaces, shape operator, Principal curvature, Gaussian curvature, Mean Curvature, Computations in Euclidean hypersurfaces
(5) Sectional Curvatures and its relations with Ricci and scalar curvature

- References:

[1] David Gilbarg, Neil S. Trudinger, Elliptic Partial Differential Equations of Second Order.

[2] Dennis Kriventsov, Course Note for Rutgers Math 518.
[3] John M. Lee, Introduction to Riemannian manifolds, second edition.
[4] Lawrence C. Evans, Partial Differntial Equations, second edition.
[5] Lawrence C. Evans, Ronald F. Gariepy, Measure Theory and Fine Properties of Functions, Revised Edition.
[6] Qing Han, An introduction to Elliptic Differential Equations.
[7] Sagun Chanillo, Course Note for Rutgers Math 518.
[8] Zheng-Chao Han, Lecture Notes of an introductory graduate PDE course.