

Oral Qualifying Exam Syllabus

Xiao Chen

1 Mathematical Finance

a. Risk-Neutral Pricing

Risk-Neutral Measure

Fundamental Theorems of Asset Pricing

b. Connections with PDE

Feynman-Kac Theorem

Black-Scholes-Merton Equation

c. Options

Knock-out barrier option

Lookback option

Asian option

American option, American perpetual put option

d. Jump Process

Poisson process, Compound Poisson process, Jump process

Ito's formula for jump process

Change of measure

Pricing a European call in jump model

e. Black and Scholes Model

Construction of self-financing strategies

Absence of arbitrage

f. Arbitrage-free Pricing in Black and Scholes Model

Black and Scholes PDE for standard European options

Risk-neutral valuation for European options

Valuation of American options

g. Optimal investment in Black and Scholes model

The Hamilton-Jacobi-Bellman equation

The martingale characterization of optimal investment strategy

2 Stochastic Calculus

a. Stochastic Process

Filtration, Stopping time, Optional time

Martingale, Submartingale, Supermartingale, Local martingale

Upcrossing inequality, Submartingale convergence theorem

Doob's maximal inequality

Optional Sampling Theorem

Doob's-Meyer Decomposition

Square-integrable Martingales, Quadratic variation

b. Brownian Motion

Brownian Motion, Construction of Brownian Motion

The invariance principle

Markov Property, Reflection Principle

Distribution of Brownian Motion and its running maximum

Distribution of first passage time

Strong law of large number for standard Brownian Motion

The law of iterated logarithm

c. Stochastic Integration

Construction of stochastic integral

Ito's rule, Martingale characterization of Brownian Motion

The Burkholder-Davis-Gundy inequalities

Girsanov Theorem, Novikov Condition

d. Stochastic Differential Equations

Strong solutions, Existence and uniqueness

Weak solutions, Existence and uniqueness

References

- [1] Steven E. Shreve, *Stochastic Calculus for Finance II*
- [2] Ioannis Karatzas, Steven E. Shreve, *Brownian Motion and Stochastic Calculus, second edition*
- [3] Darrell Duffie, *Dynamic Asset Pricing Theory*