

Oral Qualifying Exam Syllabus

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Topics: Functional Analysis, Topics in Mathematical Physics

1 Functional Analysis

1. Banach Spaces
 - (a) Metric Spaces, Normed Linear Vector Spaces
 - (b) Hahn-Banach Theorem
 - (c) Baire Category Theorem, Uniform Boundedness Principle, Open Mapping Principle, Closed Graph Theorem
 - (d) Duality, Reflexivity, Sequential Compactness, Weak Topology, Weak * Topology, Banach-Alaoglu Theorem, Riesz's lemma
 - (e) L^p Spaces, properties, reflexivity, separability, dual, Convolution and Regularization
2. Hilbert Spaces
 - (a) Definition of Hilbert Space and its Dual Space
 - (b) Bounded operators, Adjoint operators and properties
 - (c) Projection Theorem
 - (d) Riesz representation theorem
 - (e) Orthogonal sets, Bessels inequality
3. Compact Operators
 - (a) Definition, elementary properties and adjoint
 - (b) The spectrum of a compact and compact self-adjoint operator
 - (c) Fredholm Alternative
 - (d) Spectral invariance and the abstract spectral theorem in C^* algebras
 - (e) Strong and weak topologies, σ -weak topology, strong closure and related theorems, trace class operators
 - (f) Baire Functional Calculus
 - (g) Polar Decomposition

2 Topics In Mathematical Physics

1. Newtonian Point Mechanics
 - (a) Newton's equation of motion, Galilean invariance, The 2-body problem for attractive $1/r$ potential
 - (b) Lagrangian formulation: Euler-Lagrange equations, Equivalence to Newton's equation of motion, Noether's Theorem (symmetry and conservation laws)
 - (c) Hamiltonian formulation: Legendre transformations, Equivalence to Newton's equation of motion
2. Einstein's Special Relativity
 - (a) Minkowski Spacetime: Poincaré Group, 4-vectors and invariants (proper time, wave operator etc.)
 - (b) The Lorentz Force law for test particles (test particle motion in a uniform electric and magnetic field), Maxwell's equations for fields given charges/currents, Derivation of wave equation in Lorentz gauge, Gauge invariance, Covariant form of Maxwell's equations
3. Non-Relativistic Quantum Mechanics:
 - (a) Schrödinger's Equation, The (non-relativistic) hydrogen atom as a 1-body problem, N-body Schrödinger Equation
4. RESEARCH TOPIC: Quantum Entropy
 - (a) Operator Convexity and Monotonicity for Complex Matrices
 - (b) Löwner-Heinz Theorem, Klein's Inequality, Peierls-Bogoliubov Inequality, Arithmetic-Geometric-Harmonic means, Positive Weighted Geometric Mean and its extension as a geodesic
 - (c) Majorization, Log Majorization, and relation to unitarily invariant norms
 - (d) Golden-Thompson like Matrix Inequalities
 - (e) Trace Inequalities and Quantum Entropy
5. RESEARCH TOPIC: General Relativity
 - (a) General Relativity: metric, coordinates and coordinate changes
 - (b) Geodesics, affine connections, Christoffel symbols
 - (c) Vacuum Solutions to the Einstein Equations: Schwarzschild Black Holes and their properties
 - (d) Event horizons and Cauchy horizons, maximal extensions, Penrose diagrams