

ORAL QUALIFYING EXAM SYLLABUS

Vernon Chan

Algebraic Geometry

1. **Sheaves:** Presheaves and associated sheaves, morphisms of sheaves, stalks, pull-back and push-forward
2. **Schemes:** Spec and Proj, morphisms of schemes, generic points
3. **First properties of schemes:** Reduced, integral, noetherian schemes, morphisms of finite type, finite morphisms, open and closed immersions, fibre products
4. **Separated and proper morphisms:** Definition of separated and proper morphisms, valuative criteria, projective morphisms, scheme-theoretic images
5. **Coherent and quasi-coherent sheaves:** Definition of $\mathcal{O}_X$ -modules, quasi-coherent and coherent sheaves, constructions of $\mathcal{O}_X$ -modules, invertible sheaves, twisted sheaves, very ample invertible sheaves, vector bundles
6. **Divisors:** Weil divisors, Cartier divisors, Weil and Cartier divisor class groups, equivalence for locally factorial schemes, Picard groups
7. **Projective morphisms:** Criteria and characterization of projective morphisms, ample and very ample line bundles, blowing ups
8. **Differentials:** Derivations, sheaves of relative differentials, connection to nonsingularity, Bertini's theorem, tangent sheaves, canonical sheaves, geometric genus

Cohomology Theory of Sheaves

1. **Derived functors:** Abelian categories, complexes, derived functors, δ -functors
2. **Cohomology of sheaves:** The category of sheaves of $\mathcal{O}_X$ -modules has enough injectives, Grothendieck's vanishing theorem

3. **Cohomology of Noetherian Affine Schemes:** Cohomological characterization of noetherian affine schemes
4. **Čech Cohomology:** Definition, computation of sheaf cohomology groups for a noetherian separated scheme
5. **Cohomology of Projective Spaces:** properties of cohomology groups of projective schemes, criterion for ampleness
6. **Extension Groups and Sheaves**
7. **Serre Duality Theorem:** Duality for $\mathbf{P}_k^n$, dualizing sheaves, duality for projective schemes
8. **Spectral Sequences**

Toric Varieties

1. **Definitions:** Convex polyhedral cones, geometry of convex sets, affine toric varieties, fans and toric varieties, toric varieties from polytopes
2. **Singularities and compactness:** Local properties of toric varieties, surfaces, quotient singularities, one-parameter subgroups, limit points, compactness and properness, nonsingular surfaces, resolution of singularities
3. **Orbits, topology, and line bundles:** Orbits, fundamental groups and Euler characteristics, divisors, line bundles, cohomology of line bundles, canonical class
4. **Cohomology of smooth toric varieties:** Stanley-Reisner relation, self-intersection of invariant divisors

References

- [1] R. Hartshorne, *Algebraic Geometry*, Springer-Verlag, 1977.
- [2] D. Mumford, E. Arbarello, *The Red Book of Varieties and Schemes* (2 ed.), Springer, 1999.
- [3] J. Harris, *Algebraic Geometry: A First Course*, Springer, 1995.
- [4] W. Fulton, *Introduction to Toric Varieties*, Annals of Math Studies, 1993