

ORAL EXAM SYLLABUS

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1 SYMPLECTIC TOPOLOGY

Symplectic structures

Skew-symmetric bilinear forms; symplectic vector spaces, (co)isotropic and Lagrangian subspaces; symplectic manifolds and symplectomorphisms; exact symplectic structure on cotangent bundles; diffeomorphic manifolds have symplectomorphic cotangent bundles; symplectic volume invariant.

Lagrangian submanifolds

Lagrangian submanifolds; symplectic and Hamiltonian isotopies; graph and conormal Lagrangians in cotangent bundles; symplectomorphisms are diffeomorphisms with Lagrangian graph; Maslov index.

Local theory

Deformation of symplectic structures; isotopic and strongly isotopic symplectic structures; Moser trick; Darboux coordinates; Weinstein Lagrangian neighborhoods.

Contact structures

Contact structures and contact forms; Legendrian submanifolds; contact structure on projective and sphere cotangent bundles; contact Darboux coordinates; Reeb vector fields and contactomorphisms; symplectization of a contact manifold.

Almost complex structures

Existence of compatible ac structures; the set of compatible ac structures is path connected; the space of compatible ac structures is contractible; compatible symplectic structures are

deformation equivalent; almost complex submanifolds are symplectic; basic Dolbeault theory; Nijenhuis tensor and integrability of ac structures; Kähler structures and properties; global and local potentials of Kähler forms; Fubini-Study form on smooth projective varieties.

Moment maps

Hamiltonian actions and moment maps; basic properties of moment maps; Poisson structure on dual Lie algebra; Meyer/Marsden-Weinstein theorem.

J-holomorphic curves

Cauchy-Riemann equations for curves in almost complex manifolds; critical points and non-injective points, simple curves; linearization $D = D_u(\bar{\partial}_J)$ is a real CR operator; Calderón-Zygmund inequality, D has finite kernel and closed image; formal adjoint D^* has finite kernel and closed image; elliptic regularity, $\ker(D^*) = \operatorname{coker}(D)$ and $\operatorname{coker}(D^*) = \ker(D)$; C^∞ Banach manifold $\mathcal{J}^l$ of C^l almost complex structures; C^{l-k} Banach manifold $\mathcal{M}(A, \mathcal{J}^l)$ of simple curves of class A ; $\pi : \mathcal{M}(A, \mathcal{J}^l) \rightarrow \mathcal{J}^l$ is C^{l-k} , C^l regular ac structures and transversally cut out C^{l-k} moduli spaces; Morse-Smale theorem, for l large C^l regular ac structures are second Baire category in $\mathcal{J}^l$; Taubes argument, C^∞ regular ac structures are second Baire category in $\mathcal{J}$; variation of regular ac structure and cobordism of moduli spaces; energy and area match for compactible symplectic forms; elliptic bootstrapping, convergence of uniformly bounded curves; isoperimetric inequality, removal of singularities for curves with bounded energy; universal energy lower bound; conformal rescaling, bubbling of curves with bounded energy; index formula.

2 ALGEBRAIC GEOMETRY

Sheaves and schemes

Presheaves, sheaves, stalks, morphisms, sheafification; pullback and pushforward of sheaves; ringed spaces and morphisms; affine, projective and general schemes; open and closed immersions; reduced, irreducible and integral schemes; generic and special points; Krull dimension; algebraic varieties and function fields; transcendence degree; residue fields of points; closed and rational points.

Morphisms and base change

Fibered product of schemes; scheme-theoretic fiber of a morphism; finite and finite type morphisms; quasi-compact and separated morphisms; proper morphisms; affine and projective morphisms; rational maps.

Local properties

Normal schemes and normalization; tangent space and derivative of morphisms; singular points and Jacobian criterion; flat morphisms; ramification and étale morphisms.

Sheaf Cohomology

General, quasi-coherent and coherent $\mathcal{O}_X$ -modules; pullback and pushforward along qcqs

morphisms; ideal sheaves and correspondence with closed subschemes; locally free and twisted $\mathcal{O}_X$ -modules; globally generated $\mathcal{O}_X$ -modules; the category of $\mathcal{O}_X$ -mod has enough injectives; sheaf cohomology as right derived functor of global sections; sheaf cohomology from Čech complex of a covering; Leray acyclicity theorem; Grothendieck cohomological vanishing theorem; higher pushforward, Serre spectral sequence and projection formula; cohomology of $\mathcal{O}_{\mathbb{P}^n}(d)$ and Serre finiteness theorem; (very) ampleness and Serre cohomological criterion; Euler characteristic and Hilbert polynomial of $\mathcal{O}_X$ -mod.

Differentials

Algebraic differential forms; dualizing sheaf and Serre duality; canonical sheaf of a smooth projective variety is dualizing; (co)normal sheaf, regular immersions and local complete intersections; adjunction formula.

Divisors

Cartier divisors and associated invertible sheaf; multiplicity and support of Cartier divisors; Cartier divisors of rational sections of invertible sheaves; Weil divisors and relation with Cartier; Picard group.

Toric varieties

Toric varieties and morphisms; basic combinatorics of cones and fans; toric variety of fans and toric morphism of simplicial lattice maps; normality, smoothness and properness; structure of torus orbits and their closures; Weil divisors of rays; Cartier divisors of integral PL functions; sections of line bundles from lattice points of rational convex polyhedron; characterization of basepoint free and ample line bundles; sheaf cohomology of line bundles from conical subsets of fan.

REFERENCES

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- [2] Dusa McDuff, Dietmar Salamon. *Introduction to Symplectic Topology*. Oxford University Press, Graduate texts in mathematics 27, 2017.
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