

ORAL QUALIFYING EXAM SYLLABUS

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Primary Topic: Experimental Mathematics

- Basics
 - Maple and Dynamic Programming
 - Selection sort and Quicksort algorithms
 - Recurrence equations
 - Symbolic Sums
 - Generating Functions
 - Asymptotic Estimates
- Formal Power Series
 - Differentiation and Division
 - Sequences of Power Series
 - The Transfer Principle
 - Multivariate Power series
 - Truncated Power Series
- Polynomials
 - Polynomials as power series
 - Polynomials as sequences
 - The Tetrahedron for polynomials: symbolic sums, asymptotic estimates, generating functions, recurrence relations
 - Polynomials as solutions
 - Polynomials as coefficients
- C-Finite Sequences
 - Fibonacci Numbers
 - Recurrences with Constant Coefficients
 - Closure Properties
 - The Tetrahedron for C-finite sequences: symbolic sums, asymptotic estimates, generating functions, recurrence relations
 - System of C-finite Recurrences
- Hypergeometric Series
 - The Binomial Theorem
 - Basic Facts and Definitions
 - Asymptotic estimates, generating functions, recurrence relations
 - Indefinite summation

- Definite summation
- Algebraic Functions
 - Catalan numbers
 - Basic facts and definitions
 - Puiseux series and the Newton Polygon
 - Closure Properties
 - The Tetrahedron for Algebraic Functions: symbolic sums, asymptotic estimates, generating functions, recurrence relations

Reference: Kauers and Paule, *The Concrete Tetrahedron* Chapters 1-6

Secondary Topic: Algorithmic Algebraic Geometry

- Basics
 - Polynomial Rings and their ideals
 - Radicals
 - Hilbert Basis Theorem
 - Monomial orderings
 - Polynomial division in $k[x_1, \dots, x_n]$
 - Affine Varieties
- Gröbner Bases
 - Using Buchberger’s criterion and Buchberger’s algorithm
 - Gröbner basis conversion
- Solving polynomial equations
 - Elimination Theory: Extension Theorem, Elimination Theorem, the Geometry of Elimination
 - Implicitization
 - Solving equations via Eigenvalues and Eigenvectors
- Resultants
 - The resultant of two polynomials
 - Mutlipolynomial resultants
 - Properties of resultants
 - Calculating resultants
 - Solving equations via resultants

References:

Cox, Little, and O’Shea, *Using Algebraic Geometry* Chapters 1, 2 (sections 1, 3, and 4), and 3
 Cox, Little, and O’Shea, *Ideals, Varieties, and Algorithms* Chapters 1-4