Oral Exam Syllabus

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Primary Topic: Experimental Math

1.1 Enumeration

- Maple coding, dynamic programming
- Generating functions, Lagrange Inversion Formula
- Goulden-Jackson Cluster Method
- Cayley's Formula for Labelled Trees

1.2 Symmetric Functions

- Elementary, Monomial, Homogenous bases for symmetric polynomials
- Young Tableaux, RSK bijection (statement)
- Hook-Length formula (and proof)
- Schur Polynomials (combinatorial and algebraic definitions)

Secondary Topic: Combinatorics and Graph Theory

2.1 Combinatorics

- Basics: binomial coefficients and identities, inclusion-exclusion, Stirling's formula
- Hypergraphs: Erdos-Ko-Rado, Fisher's Inequality, Ray-Chaudhuri Wilson (uniform and non-uniform)
- Correlation Inequalities: Harris-Kleitman, FKG, Ahlswede-Daykin
- Ramsey Theory: Ramsey's Theorem for graphs and hypergraphs, infinite Ramsey theorem, upper and lower bounds, van der Waerden's Theorem, Schur's Theorem
- Polynomial Methods: Combinatorial Nullstellansatz, Cauchy-Davenport Theorem, Schwartz-Zippel Lemma, Erdös-Heilbronn Conjecture
- Misc: Beck-Fiala

2.2 Probabilistic Methods

- Linearity of expectation, Union bound, Bonferroni Inequalities, Markov Inequality, Chebyshev's Inequality, Chernoff Bounds

- Method of Alterations: Graphs with high girth and high chromatic number, Ramsey lower bounds, independence number of graphs
- Second Moment Method: Threshold to contain a fixed subgraph
- Lovasz Local Lemma, symmetric and general version, application to Ramsey lower bounds
- Janson's Inequality, triangles in $G_{n,p}$
- Entropy: basic properties, Shearer's Lemma, Bregman's Theorem

2.3 Graph Theory

- Matchings: Hall's Theorem, Konig's Theorem, Tutte's Theorem
- Chains and Antichains: Sperner's Theorem, LYM Inequality, Dilworth's Theorem,
- Planar Graphs: Euler's formula, Kuratowski's Theorem
- Colorings: Five Color Theorem, Brooks' Theorem, Vizing's Theorem
- Extremal: Turan's Theorem, Szemeredi Regularity Lemma (statement), Erdos-Stone Theorem, Triangle Removal Lemma, Roth's Theorem
- Games on Graphs: Erdos-Selfridge, Hales-Jewett