

SYLLABUS FOR ORAL QUALIFYING EXAM

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I. Major Topic: Partial Differential Equations

1. Laplacian Equation

- (1) Fundamental Solutions.
- (2) Mean value formulas and converse to mean-value property
- (3) Properties of harmonic functions:
Maximum principle, smoothness, local estimates, Liouville's theorem, analyticity, Harnack's inequality, removable singularity, Schwarz reflection principle.
- (4) Green's functions for a ball and for a half-space.
- (5) The classical Dirichlet problem by Perron's method.
- (6) The energy estimates

2. Heat Equation

- (1) Fundamental Solutions.
- (2) Maximum principle on bounded $\overline{U}_T$ and $\mathbb{R}^n \times [0, T]$ the uniqueness by maximum principle.
- (3) Uniqueness and backward uniqueness by energy method.

3. Wave equation

- (1) Solutions by spherical means for dimension 1, 2 and 3:
d'Alembert's formula for dimension 1, Kirchhoff's formula for dimension 3 and Poisson's formula for dimension 2.
- (2) Uniqueness and domain of dependence by energy methods

4. Sobolev Space

- (1) Definition of Sobolev space.
- (2) Approximation by smooth functions
- (3) Extensions and traces
- (4) Sobolev inequalities:
Gagliardo-Nirenberg-Sobolev inequality, Morrey's inequality.
- (5) Compact embedding theorem: Rellich-Kondrachov theorem.

- (6) Poincare's inequality, Difference quotients.

5. Second Order Elliptic Equations

- (1) Definition of weak solutions
- (2) Existence by Lax-Milgram theorem and energy estimates by Fredholm alternative.
- (3) Interior regularity and boundary regularity
- (4) Weak and strong maximum principles, Hopf's lemma.
- (5) Harnack inequality

II. Minor Topic: Functional Analysis

1. Banach Spaces

- (1) Metric spaces, normed linear vector spaces, Banach spaces and Hahn-Banach theorem.
- (2) The Baire category theorem, the uniform boundedness principle, the open mapping principle and the closed graph theorem.
- (3) Duality, reflexivity, weak topology and weak* topology.

2. L^p Spaces

- (1) Definition and Elementary Properties of L^p Spaces
- (2) Reflexivity. Separability. Dual of L^p
- (3) Convolution and regularization
- (4) Criterion for Strong Compactness in L^p

3. Hilbert Spaces

- (1) Definition of Hilbert space and the dual space of Hilbert space.
- (2) The Riesz representation theorem and the Lax-Milgram theorem
- (3) Orthogonal sets, Basis, the Bessel's inequality and the Parseval's theorem.

4. Compact Operator

- (1) Definition, elementary properties and adjoint.
- (2) The Riesz-Fredholm theory.
- (3) The spectrum of a compact operator

References

- [1] Lawrence C. Evans, *Partial Differential Equations*, AMS, 1998.
- [2] David Gilbarg, Neil S. Trudinger, *Elliptic Partial Differential Equations of Second Order*, Springer, 2001.
- [3] Haim Brezis, *Functional Analysis, Sobolev Spaces and Partial Differential Equations*, Springer, 2010.