

# Oral Qualifying Exam Syllabus

Minhao Bai

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## 1 Major Topic: Combinatorics

### 1.1 Sets

- Enumeration: Counting and double counting, pigeonhole principle, inclusion-exclusion, Stirling's formula, generating function, binomial identities, Catalan numbers.
- Set systems: Sperner's theorem, LYM inequality, uniform and non-uniform Ray-Chaudhuri-Wilson theorem, Erdos-Ko-Rado Theorem, Fisher's inequality, Kruskal-Katona theorem, Frankl-Wilson theorem.
- Posets: Dilworth's theorem, Birkhoff representation theorem, linear extension.
- Ramsey Theory: Ramsey's theorem, infinite Ramsey, van der Waerden's theorem. Hale-Jewett theorem, Rado's Theorem, the compactness principle.

### 1.2 Graph Theory

- Basics, Graph invariants.
- Spanning Tree and algorithm.
- Flow and cut: Menger's theorem, Max-flow-min-cut Theorem, related algorithm.
- Matching: Hall's theorem, Konig's theorem, Tutte's 1-factor theorem.
- Coloring: Brook's theorem, Vizing's theorem, Galvin's theorem, weak perfect graph theorem.
- Planarity: Kuratowski's theorem, Wagner's theorem.
- Extremal: Turan's theorem, Erdos-Stone theorem, Szemerédi regularity lemma.

### 1.3 Probabilistic Method

- Basics: Linearity of expectation, law of total probability, Bonferroni's, Markov's, Chernoff's inequalities.
- Second moment method: Variance, Chebyshev's inequality, application to thresholds.
- Lovasz local lemma
- Correlation inequalities: FKG, Harris
- Poisson paradigm
- Entropy: basics, inequality, entropy method.
- Martingale: Azuma's inequality, vertex and edge exposures.

## 2 Minor Topic: Complexity Theory

- Basics: P vs NP, NP-Completeness.
- Interactive proofs,  $IP=PSPACE$
- PCP Theorem
- Hardness of Approximation, Unique Games Conjecture