

Oral Qualifying Exam Syllabus

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Analytic Number Theory

1. Poisson summation. Mellin transform.
2. Riemann Zeta function and Dirichlet L-functions. Their analytic continuation, Euler product, Hadamard product, explicit formula, and convexity bound.
3. Dirichlet's theorem on primes in arithmetic progressions. Landau lemma.
4. Prime number theorem for primes in arithmetic progressions.
5. p -adics. Ostrowski's theorem. Local fields.
6. Tate's thesis. Adeles, ideles.

Modular Forms

1. Modular forms and cusp forms. Fourier expansion, especially for Eisenstein series. Dimension of $M_k(\Gamma)$ for $\Gamma = SL(2, \mathbb{Z})$.
2. The Jacobi θ -function, Δ -function, and j -function.
3. L -functions of modular forms and functional equations.
4. Hecke operators. Commutativity and self-adjointness with respect to Petersson inner product.
5. Non-holomorphic Eisenstein series.

Representation Theory

1. Representations of finite groups.

2. Finite and infinite dimensional representations of $sl(2, \mathbb{R})$. Unitary dual of $SL(2, \mathbb{R})$.
3. Complex representations of $GL(2, \mathbb{F}_q)$.
4. Lie algebras. Roots, weight spaces, Killing form, Weyl group, simple roots.