Oral Qualifying Exam Syllabus

Maxwell Aires

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1 Main Topic: Combinatorics

1. **Enumeration:** Counting and double counting, Pigeonhole principle, Inclusion-exclusion, Stirling's Approximation/Asymptotics, Binomial Identities, Generating Functions, Recursive Relations
2. **Set Systems:** Sperner's Theorem, LYM Inequality, Kruskal-Katona Theorem, Ray-Chaudhuri-Wilson Theorem, Erdos-Ko-Rado Theorem, Sauer-Shelah Theorem, Frankl-Wilson Theorem
3. **Probabilistic Method:** First and Second Moment Methods, Markov and Chebyschev Inequalities, Chernoff Inequalities, Lovasz Local Lemma, Janson's Inequality, Thresholds for containing a fixed graph/isolated point, Martingales and Azuma's Inequality, Basic properties of Entropy/Shearer's Inequality
4. **Graph Theory:**
   - **Connectivity and Flow** Menger's Theorem (multiple forms), Max-Flow-Min-Cut Theorem, Structure of 2-connected graphs
   - **Algorithms** Prim, Kruskal, Djikstra
   - **Matchings** Hall's Theorem, Tutte's Theorem, Konig's Theorem
   - **Coloring** Existence of graphs with high girth/chromatic number, Brooks' Theorem, Vizing's Theorem, Galvin's Theorem for Bipartite Graphs, Basic properties of perfect graphs, Low List-colorability implies low average degree
   - **Planarity** Erdos's Formula, Kuratowski's Theorem, Wagner's Theorems, Five-List-Colouring of Planar graphs
5. **Extremal Combinatorics and Ramsey Theory:** Ramsey's Theorem (Finite and Infinite), Bounds on Ramsey Numbers, Turan's Theorem, Erdos-Stone Theorem, Hales-Jewett, van der Waerden, Erdos-Szekeres, Regularity Lemma, Triangle-Removal Lemma
6. **Poset Theory:** Dilworth's Theorem, Birkhoff Representation Theorem, Correlation Inequalities (Harris, FKG, Four-Function), Linear Extensions and xyz Inequality
7. **Algebraic Methods:** Combinatorial Nullstellenzatz, Schwartz-Zippel Lemma, Linear-Algebraic methods
8. **Additive Combinatorics:** Ruzsa Triangle Inequalities, Plünnecke Inequality, Balog-Szemeredi-Gowers Theorem, Elekes' bound for Sum/Product, Roth's Theorem, Behrend's construction for avoiding 3-arithmetic sequences
9. **Combinatorial Geometry** Helly's Theorem, Szemeredi-Trotter Theorem, Brunn-Minkowski Theorem

2 Minor Topic: Complexity Theory

1. **P vs NP:** Polynomial Reductions, Cook-Levin Theorem, Various NP-complete problems (SAT, 3SAT, Independent Sets)
2. **Space Complexity:** PSPACE-complete problems (TQBF), NL-completeness of PATH, Savitch's Theorem, Immerman-Szelepcsenyi Theorem, IP = PSPACE
3. **Diagonalization:** Time and Space hierarchy theorems (deterministic and nondeterministic), Ladner's Theorem
4. **Circuits** Cook-Levin vis CIRCUIT-SAT, Karp-Lipton Theorem, Nonuniform Hierarchy theorem, Definitions of AC^i, NC^i, Hastad's switching lemma and $\bigoplus \notin AC^0$ (and MOD_m generalization), Lowerbound for monotone circuits computing $\text{CLIQUE}_{k,n}$
5. **Polynomial Hierarchy:** Σ_i, Π_i, Conditions leading to the collapse of PH
6. **Randomization:** RP, BPP, ZPP, BPP $\subseteq$ PH, Error-Reduction, BPP $\subseteq$ P/poly, Valiant–Vazirani Theorem
7. **Decision Tree Complexity:** Decision tree complexity (deterministic and random), Yao's Lemma, degree, sensitivity, relationships between quantities