Book #1 of 1

Name: ___________________________

ID# (last 4 digits): ___________________  Section: _______________________

Unless stated otherwise, you must show all work clearly using proper notation and explain your reasoning in English where appropriate. Answers must be justified using techniques that have been taught in this course, and answers without such justification may receive less than full credit – or no credit at all – even if the answer is correct.

Some problems (e.g., multiple-choice, true-false, fill-in, etc.) may be marked as “no partial credit”. For these problems, you are not required to show work, and any scratch work will not be considered. You will be awarded none or all of the points, depending only on whether your answer is exactly correct.

This exam is closed book. Calculators, electronic devices, notes, books, formula sheets, and other outside materials are not allowed. Phones must be turned off and put away.

Unless otherwise stated, give exact answers: e.g., write π and √2 instead of 3.14 and 1.41. However, when an expression simplifies to a well-known value, you must use that value. For example, you must write 1 instead of $e^0$, and you must write $\frac{1}{2}$ instead of $\cos\left(\frac{\pi}{3}\right)$.

You must justify all uses of L'Hospital's Rule (LR). If you use LR for any calculation, you must indicate why LR is applicable. It is also preferred, but not necessary, that you use the symbol $\Rightarrow$ instead of a normal equals sign to indicate the exact step in which you use LR.

This exam has 7 questions, printed in 1 booklet(s), for a total of 100 points.

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1. For each part, write your answer on the provided line. You are not required to show work, but you may use the provided space for scratch work. For each part, there is no partial credit.

(a) Find the slope of the tangent line to the curve $x^3 - y^3 = y - 1$ at the point $(1, 1)$.

slope of tangent line: ________________

(b) The total revenue from selling $x$ units of a certain product is $R(x) = 40 - \frac{200}{x + 5}$. Using marginal analysis, estimate the revenue from selling the 6th unit.

revenue from 6th unit $\approx$ ________________
(c) Calculate the absolute minimum value of \( f(x) = x^3 - 3x \) on the interval \([0, 2]\).

absolute minimum value: 

(d) Use a linear approximation to estimate the value of \((16.32)^{1/4}\).

\((16.32)^{1/4} \approx \)
(e) Calculate the derivative of $f(x) = x^x$. Your final answer must contain only $x$.

$$f'(x) =$$

(f) Find the equation of each horizontal asymptote of $f(x) = \frac{2e^x - 5}{3e^x + 2}$. Write "NONE" as your answer if appropriate.

$$\text{equation(s) of horizontal asymptote(s):}$$
2. Consider the function  

\[ f(x) = e^{-x^2/2} \]

Find where \( f \) is concave down and find where \( f \) is concave up. Then find all inflection points (\( x \)- and \( y \)-coordinates). Write “NONE” for your answer if appropriate.

where \( f \) is concave down: ________________________

where \( f \) is concave up: ________________________

inflection point(s): ________________________
3. Consider the function

\[ f(x) = \frac{1}{x^2 - 6x} \]

Find all vertical asymptotes of \( f \). Then find where \( f \) is decreasing and find where \( f \) is increasing. Finally determine the \( x \)-coordinates of all local extrema of \( f \) (and classify them as either a local minimum or a local maximum). Write “NONE” for your answer if appropriate.

vertical asymptote(s): __________________________

where \( f \) is decreasing: __________________________

where \( f \) is increasing: __________________________

\( x \)-coordinate(s) of local minima: __________________________

\( x \)-coordinate(s) of local maxima: __________________________
4. The surface area of a sphere is changing at a rate of $16\pi \text{ in}^2/\text{s}$ when its radius is 3 in. At what rate is the volume of the sphere changing at that time?

*You must include correct units as part of your answer.*

*Hint:* If a sphere has radius $R$, then its surface area $A$ and volume $V$ are given by

$$A = 4\pi R^2, \quad V = \frac{4\pi}{3} R^3$$

rate of change of volume: ____________
A poster is to have a total area of 150 in\(^2\), which includes a central printed area, 1-inch margins at the bottom and sides, and a 2-inch margin at the top. What poster dimensions (in inches) will give the largest printed area? Use calculus to justify your answer.

**You must demonstrate that your answers really are the optimal dimensions.**

optimal width: \(x=\) ______________

optimal height: \(y=\) ______________
6. For each part, calculate the limit or show that it does not exist. If the limit is infinite, write “∞” or “−∞” as your answer, as appropriate.

(a) \[ \lim_{x \to 3^-} \frac{x^2 + 6}{3 - x} \]

value of limit (or write “DNE”): ____________________

(b) \[ \lim_{x \to 0} (1 - \sin(3x))^{1/x} \]

value of limit (or write “DNE”): ____________________
7. This problem asks about the Mean Value Theorem (MVT).

(a) Explain precisely why the function $f(x) = 10 - x^{2/5}$ does not satisfy the hypotheses of the MVT on the interval $[-1, 1]$. 

(b) The function $f(x) = x^3$ satisfies the hypotheses of the MVT on the interval $[1, 3]$. Find all values of $c$ referenced in and guaranteed to exist by the MVT.

all guaranteed values of $c$: _________________