MATH 111

Review Exercise for the Math 111 Final Exam

The following are review exercises for the Math 111 final exam. These exercises are provided for you to practice or test yourself for readiness for the final exam. While preparing for the exam, students should not only rely on this review material. To best prepare, students may want to review also their recitation quizzes and applicable parts of Chapter 1, 2, and 3 review exercises from the textbook. There are many more problems appearing here than would be on the final. These exercises represent many of the types of problems you would be expected to solve on the final, but are not meant to represent all possible types of problems that could appear on the final exam.

Your final exam will be one part. You may use only a scientific calculator for the exam, but all answers must be exact.

Unless otherwise stated, give EXACT answers: e.g., write $\frac{1}{3}, \frac{1}{2}$ (instead of 0.33, 0.5 respectively) or $\sqrt{2}$ (instead of 1.41).

Show all your work: unsupported results may not receive credit.

1. Factor completely.

   (a) $(x-1)(x+2)^2 - (x-1)^2(x+2)$

   (b) $(a^2+1)^2 - 7(a^2+1) + 10$

   (c) $3x^{-1/2} + 4x^{1/2} + x^{3/2}$

   (d) $(x-1)^{7/2} - (x-1)^{3/2}$

   (e) $x^{-1/2}(x+1)^{1/2} + x^{1/2}(x+1)^{-1/2}$

2. Perform the operations and express your answer in simplest form with positive exponents only:

   (a) $x^{-5}\left(\frac{4x^{-2}y^3}{3x^{-5}}\right)^{-2}$

   (b) $2y^3\left(\frac{12x^3y^{-2}}{3x^{-5}y^4}\right)^{1/3}$

   (c) $\frac{3(1+x)^{1/2} - x(1+x)^{-1/2}}{1+x}$

3. Perform the operations and express your answer in the simplest form.

   (a) $\dfrac{x-5}{x^2-2x-8} - \dfrac{x+1}{x+2}$

   (b) $\dfrac{x-4}{3x^3-27x} - \dfrac{2x+1}{x-3}$

4. Express the following as a simple fraction reduced to lowest terms: $\dfrac{1-\dfrac{4}{z}+\dfrac{4}{z^2}}{\dfrac{1}{z^2}-\dfrac{2}{z^3}}$

5. Solve for y in the equation: $\dfrac{1}{y}+\dfrac{1}{x}=\dfrac{1}{z}$

6. Simplify the radical: (LEAVE IN RADICAL FORM)

   (a) $\sqrt[3]{54x^5y^9z^4}$

   (b) $\sqrt{24x^3y^2z^5}\sqrt{12xyz^2}$

   (c) $\sqrt[3]{x^2}\sqrt{x}$

7. Rationalize the denominator and simplify:

   (a) $\dfrac{8x}{\sqrt[3]{2xy^2}}$

   (b) $\dfrac{6}{\sqrt{10}-\sqrt{7}}$

   (c) $\dfrac{x-3}{\sqrt{x}-\sqrt{3}}$

   (d) $\dfrac{3h}{\sqrt{x-h}-\sqrt{x}}$

8. Solve for *exact* real value(s) of x:

   (a) $\dfrac{4}{x-1}+\dfrac{2}{x+1}=\dfrac{35}{x^2-1}$

   (b) $x^{4/3}+8=0$

   (c) $6y^2-12y=-3$

   (d) $x^4-2x^2-24=0$

   (e) $\dfrac{2}{3}|2x-3|+4=10$

9. Solve the inequalities. Express your solution using interval notation. Must show complete analysis where applicable.

   (a) $3|3x-2|+1<22$

   (b) $|3x-4|-1\geq 2$

   (c) $2x^2-13x>-15$

   (d) $\dfrac{3}{x-1}\leq\dfrac{2}{x+2}$

10. An entertainment system was purchased for \$3,000 in 1997. A linear relationship was used to determine that the value of the system in 1999 was \$1,500.

    (a) Express the value, v, as a function of time, t, in years.

    (b) Use your function to predict how much would you expect the value to be in the year 2000.

    (c) Use your function to determine in which year the entertainment system will be worth nothing.

11. Given the points $A(3,1), B(-1,5), C(4,5)$ and $D(2,7)$. M is the midpoint of AB and N is the midpoint of CD.

    (a) Find the distance between M and N.

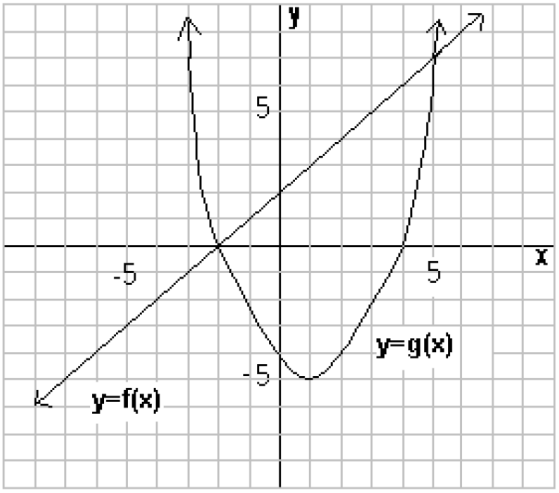
    (b) What is the equation of the circle having the center M and radius 5?

    (c) What is the equation of a circle having a center of A, and containing the point B?

    (d) What is the equation of a circle with a diameter having endpoints C and D?

12. Jason has invested in two investments: a CD that earns 4% per year and a bond that earns 8% per year. He invested \$3,520 more in the CD than the bond. How much did he invest in the CD and the bond if he receives a total of \$1,100.80 in annual interest from his two investments?

13. The graph of $f(x)$ and $g(x)$ are shown.



(a) Evaluate $(f+g)(2)$.

(b) Evaluate $f(g(1))$.

(c) Solve $f(x) = g(x)$.

(d) Find the interval(s) where $f(x) > g(x)$

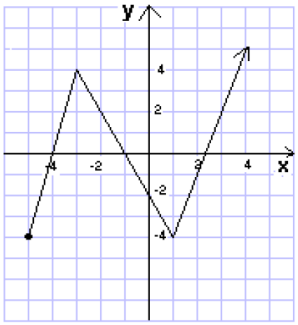
(e) Find $f^{-1}(6)$

14. Given $f(x)$ defined by the table below left, and $g(x)$ defined by the graph below right:

$f(x)$ defined by

x	f(x)
−1	3
0	1
1	−2
2	−3

$g(x)$ defined by the graph below:



(a) Evaluate $f(2)$

(b) Evaluate $(f \circ g)(2)$

(c) $g(f(1))$

15. Given $f(x) = \dfrac{4}{x}$, $\quad g(x) = \sqrt{x+4}$, find the following.

(a) Domain of $(f+g)(x)$. Express your answer using interval notaion.

(b) $(f \circ g)(x)$ and its domain.

(c) $(g \circ f)(x)$ and its domain.

16. Given $f(x) = 2x^2 - x + 1$

    (a) Evaluate $f(-2)$

    (b) Find and simplify $\dfrac{f(x+h) - f(x)}{h}$

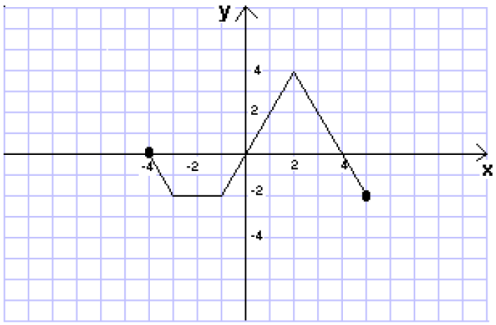
17. Given $f(x) = \dfrac{5x}{x+1}$, find and simplify $\dfrac{f(x+h) - f(x)}{h}$

18. Find the DOMAIN of the following functions: express your answer using *interval notation*. You must show your analysis to receive full credit.

    (a) $f(x) = \sqrt{4 - x^2}$

    (b) $f(x) = \sqrt{\dfrac{x+3}{x-4}}$

19. The graph of $y = f(x)$ appears below.



    (a) Sketch the graph of $f(x-2)+1$ on the same set of axes as above.

    (b) Sketch the graph of $-f(x)+2$ on the same set of axes as above.

    (c) If $h(x) = f(x-2)+1$, then evaluate $h(3)$.

20. Given $f(x) = -(x-1)^2 + 2$

    (a) Sketch the graph of $f(x)$.

    (b) Find the domain of $f(x)$.

    (c) Find the range of $f(x)$.

    (d) Find the *exact* value(s) of $x-$ intercept(s) if they exists.

    (e) Find the *exact* value(s) $y-$ intercept(s) if they exists.

    (f) Find the coordinates of the vertex.

21. Given $f(x) = \sqrt{x+1} - 2$

(a) Sketch the graph of $f(x)$.

(b) Find the domain of $f(x)$.

(c) Find the range of $f(x)$.

(d) Find the *exact* value(s) of $x-$ intercept(s) if they exists.

(e) Find the *exact* value(s) $y-$ intercept(s) if they exists.

22. Given $f(x) = \begin{cases} 2x+1 & if \quad x \leq -1 \\ 3-x^2 & if \quad x > -1 \end{cases}$

(a) Evaluate $f(2)$.

(b) Evaluate $f(-2)$.

(c) Sketch the graph of $f(x)$.

(d) Find the interval(s) where $f(x)$ is increasing.

23. Given $f(x) = \begin{cases} x+1 & if \quad x < 0 \\ x^2-1 & if \quad 0 \leq x < 2 \\ 4 & if \quad x \geq 2 \end{cases}$

(a) Evaluate $f(-2), f(0), f(1), f(2), f(10)$.

(b) Sketch the graph of $f(x)$.

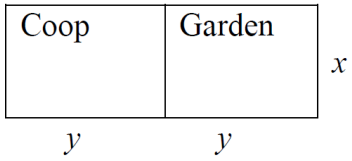
24. In economics, the demand function for a given item indicates how the price per unit p (in dollars) is related to the number of units x that are sold. Suppose that a company finds that the demand function for one of the items it produces is $p = 20 - 0.05x$.

(a) The revenue function, $R(x)$, is found by multiplying the price per unit p and number of units sold x: that is $R(x) = px$. Write the formula for the revenue function in terms of x.

(b) How many units should be produced in order to get the maximum revenue? (Show your work.)

(c) What is the corresponding unit price for your answer in part (b) above? (Show your work.)

25. A farmer wants to fence in an area for his chicken coop and for a small garden as shown in the diagram below with exactly 200 linear feet of fencing as shown in the diagram, with width x.

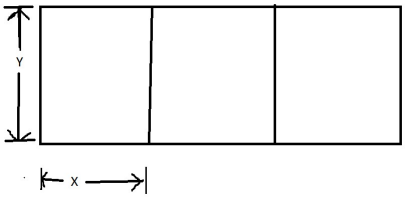


(a) Express the area of the coop A, as a function of x.

(b) Find the value of x which will give the rectangular garden with the greatest area.

(c) What is the maximum area?

26. A farmer has 1200 yards of fencing and wants to fence three identical rectangular pens that share a common fence. Let x be the length of each pen and let y be the width of each pen.



(a) Find an expression for $A(x)$, the area of the entire field, as a function of x.

(b) Find the dimensions of the rectangular field of the largest area that can be fenced.

(c) What is the maximum area?

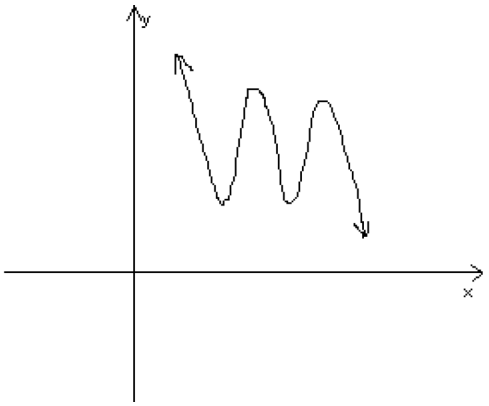
27. Given $f(x) = (x+1)(x-2)(x+3)$

(a) Graph the polynomial.

(b) What is the degree of the function?

(c) Describe the end behavior.

28. The graph of the polynomial function $y = f(x)$ is given below.



(a) What is the smallest possible degree of $f(x)$?

(b) Is it possible for $f(x)$ to have degree 8? Explain your answer.

(c) From the above graph what can you tell about the leading coefficient of $f(x)$?

29. Sketch of the graph of the following functions. Identify the $x-$intercept(s). Identify the $y-$intercept(s). Identify the horizontal asymptote(s). Identify the vertical asymptote(s).

(a) $f(x) = \dfrac{x-1}{(x-2)(x+3)}$

(b) $f(x) = \dfrac{2x^2}{x^2-9}$

30. Given $f(x) = \dfrac{x+4}{3-2x}$

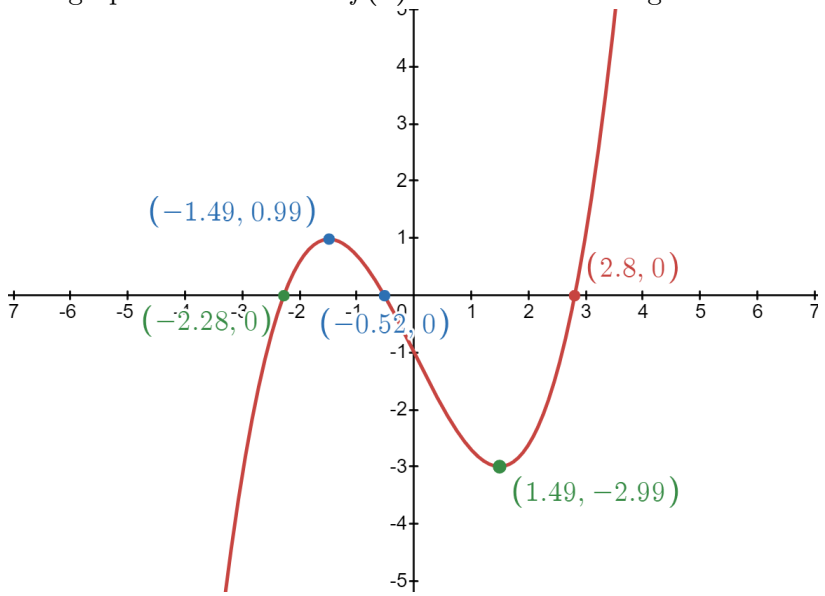
(a) Find $f(3)$

(b) Find $f^{-1}(3)$

(c) Find $f^{-1}(x)$

(d) Find $(f \circ f^{-1})(3)$

31. The graph of the function $f(x) = 0.3x^3 - 2x - 1$ is given below.



(a) Identify all zeros of $f(x)$

(b) Solve $f(x) > 0$. Express your answer using interval notation.

(c) Find the local maximum of the function $f(x)$

(d) Find the local minimum of the function $f(x)$

32. Given $f(x) = (4 - 3x)^{1/3} - 2$, find the *exact* value(s) of $f^{-1}(3)$

33. Find the equation of $g(x)$ which is obtained by performing the indicated transformations to the graph of $f(x)$.

(a) Shift the graph of $f(x)$ horizontally 2 units to the left, reflect about the $x-$ axis and stretch the graph vertically by a factor of 5.

(b) Stretch the graph of $f(x)$ horizontally by a factor of 3, move 4 units down.

ANSWERS: MATH 111 FINAL EXAM REVIEW EXERCISES

1. (a) $3(x-1)(x+2)$ (b) $(a-2)(a+2)(a-1)(a+1)$ (c) $x^{-1/2}(x+3)(x+1)$
   (d) $x(x-1)^{3/2}(x-2)$ (e) $x^{-1/2}(x+1)^{-1/2}(2x+1)$

2. (a) $\dfrac{9}{16x^{11}y^6}$ (b) $2^{5/3}x^{8/3}y$ (c) $\dfrac{3+2x}{(1+x)^{3/2}}$

3. (a) $\dfrac{-x^2+4x-1}{(x-4)(x+2)}$ (b) $\dfrac{-6x^3-21x^2-8x-4}{3x(x-3)(x+3)}$

4. $z(z-2)$

5. $\dfrac{xz}{x-z}$

6. (a) $3xy^3z\sqrt[3]{2x^2z}$ (b) $12x^2yz^3\sqrt{2yz}$ (c) $x\sqrt[6]{x}$

7. (a) $\dfrac{4\sqrt[3]{4x^2y}}{y}$ (b) $2\sqrt{10}+2\sqrt{7}$ (c) $\sqrt{x}+\sqrt{3}$ (d) $-3\sqrt{x-h}-3\sqrt{x}$

8. (a) $x=\dfrac{11}{2}$ (b) ϕ (c) $y=\dfrac{2\pm\sqrt{2}}{2}$ (d) $x=\pm\sqrt{6}$ (e) $x=-3,6$

9. (a) $\left(-\dfrac{5}{3},3\right)$ (b) $\left(-\infty,\dfrac{1}{3}\right]\cup\left[\dfrac{7}{3},\infty\right)$ (c) $\left(-\infty,\dfrac{3}{2}\right)\cup(5,\infty)$ (d) $(-\infty,-8]\cup(-2,1)$

10. (a) $v=-750t+1,500,750$ (b) \$750 (c) 2001

11. (a) $\sqrt{13}$ (b) $(x-1)^2+(y-3)^2=25$
    (c) $(x-3)^2+(y-1)^2=32$ (d) $(x-3)^2+(y-6)^2=2$

12. \$11,520 invested in the CD; \$8,000 invested in the bond.

13. (a) 0 (b) -3 (c) $x=-2,5$
    (d) $(-2,5)$ (e) 4

14. (a) -3 (b) 3 (c) 2

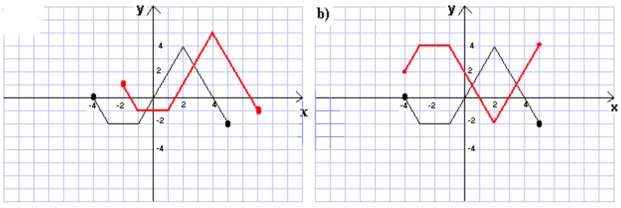
15. (a) $[-4,0)\cup(0,\infty)$ (b) $(f\circ g)(x)=\dfrac{4}{\sqrt{x+4}}$ Domain: $(-4,\infty)$
    (c) $(g\circ f)(x)=\sqrt{\dfrac{4+4x}{x}}$ Domain: $(-\infty,-1]\cup(0,\infty)$

16. (a) 11 (b) $4x+2h-1$

17. $\dfrac{5}{(x+h+1)(x+1)}$

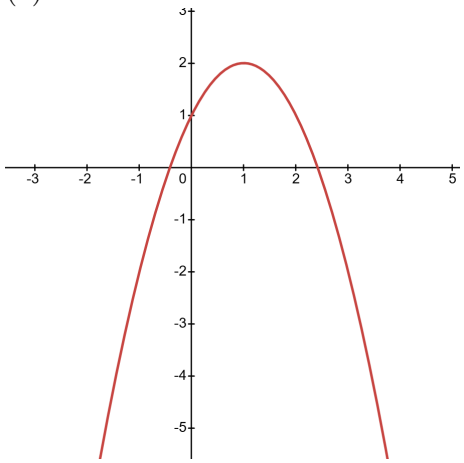
18. (a) $[-2,2]$ (b) $(-\infty,-3]\cup(4,\infty)$

19. (a)



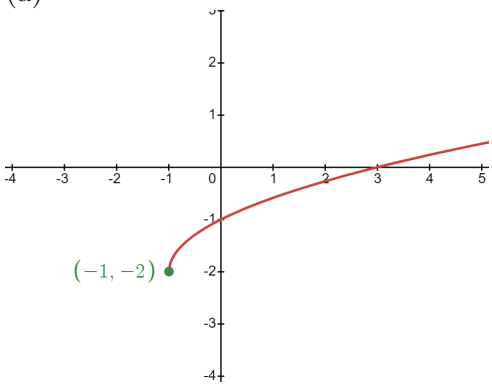
(c) 3

20. (a)



(b) $(-\infty, \infty)$ (c) $(-\infty, 2]$ (d) $1 \pm \sqrt{2}$ (e) 1 (f) $(1, 2)$

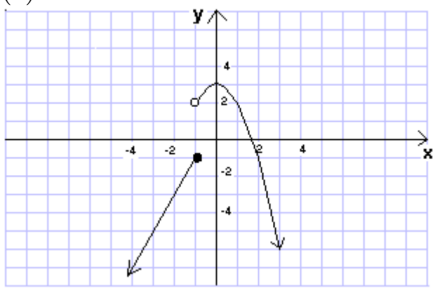
21. (a)



(b) $[-1, \infty)$ (c) $[-2, \infty)$ (d) 3 (e) -1

22. (a) -1 (b) -3

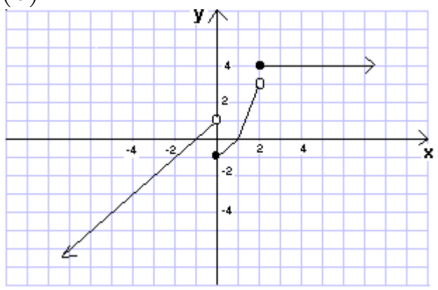
(c)



(d) Increasing: $(-\infty, 0)$

23. (a) $-1, -1, 0, 4, 4$

(b)



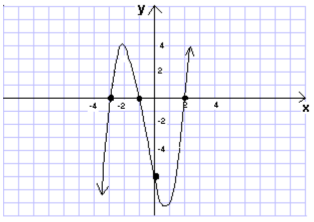
24. (a) $R(x) = (20 - 0.05x)x = -0.05x^2 + 20x$ (b) 200 units (c) \$10

25. (a) $A = \frac{1}{4}x(200 - 3x) = -\frac{3}{4}x^2 + 50x$ (b) $33\frac{1}{3}$ ft (c) $833\frac{1}{3}$

26. (a) $A = 3x\left(\frac{1200 - 6x}{4}\right) = -\frac{9}{2}x^2 + 900x$ (b) length =100 yds. width = 150 yds.

(c) Area= 45,000 square yds.

27. (a)

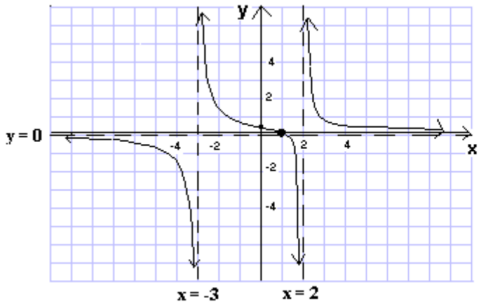


(b) 3 (c) As $x \to \infty$, $y \to \infty$ and As $x \to -\infty$, $y \to -\infty$

28. (a) 5 (b) No, the end behavior indicates that the function has odd degree

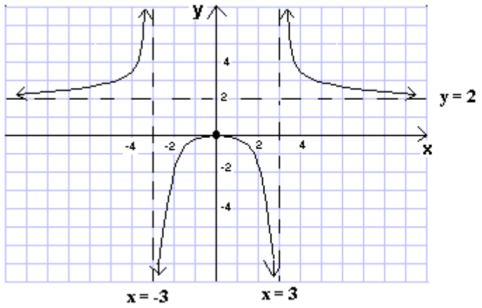
(c) The leading coefficient must be negative.

29. (a)



- $x-$ intercept $=1$, $y-$intercept$=\dfrac{1}{6}$
- Horizontal asymptote: $y = 0$
- Vertical asymptote: $x = -3, \quad x = 2$

(b)



- $x-$ intercept $=0$, $y-$intercept$=0$
- Horizontal asymptote: $y = 2$
- Vertical asymptote: $x = -3, \quad x = 3$

30. (a) $-\dfrac{7}{3}$ (b) $\dfrac{5}{7}$ (c) $f^{-1}(x) = \dfrac{3x-4}{2x+1}$ (d) 3

31. (a) $-2.28, -0.52, 2.8$ (b) $(-2.28, -0.52) \cup (2.8, \infty)$ (c) 0.99 (d) -2.99

32. $-\dfrac{121}{3}$

33. (a) $g(x) = -5f(x+2)$ (b) $g(x) = f\left(\dfrac{1}{3}x\right) - 4$