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Orthogonal polynomial expansions for the Riemann xi function

I will discuss new and old infinite series expansions for the Riemann xi function, including the Taylor series around the symmetry point; the expansion in Hermite polynomials, proposed by Turan in the 1950s as an expansion that is better suited than the Taylor series for gaining insight into whether the zeros of the function lie on a line; and two new expansions in less well-known orthogonal polynomial systems, the Meixner–Pollaczek polynomials with parameters $\phi = \pi/2, \lambda = 3/4$ and the continuous Hahn polynomials with parameters $a = b = c = d = 3/4$. These expansions arise naturally in connection with Mellin transform representations of the Riemann xi function, and the expansion coefficients exhibit good behavior, which includes alternating signs and a well-behaved rate of asymptotic decay. They also suggest natural ways of defining "flows", or one-parameter families of deformations of the Riemann xi function, which in the case of the Hermite expansion turns out to coincide with a family of deformations studied in well-known work by Polya, De Bruijn, Newman, and others.