The following problems represent some of the problems and concepts that may be on the exam. This list is not meant to be exhaustive. Thus, I would highly recommend you study all textbook problems, mixed review problems, homework problems, notes, and lectures.

Chapter 1: the Mathematics of Voting:

1. The following question pertains to an election which is held to select the president of an organization. The candidates are Ann (A), Bob (B), Claire (C), and Dave (D), and the resulting preference schedule is as follows:

# of voters:	3	5	9	4	2
1st choice	A	C	B	A	D
2nd choice	C	D	D	D	B
3rd choice	D	A	C	C	C
4th choice	B	B	A	B	A

- **a.** How many people voted in this election?
- **b.** Is there a majority candidate in this election?
- **c.** Is there a Condorcet candidate in this election?
- **d.** Find the winner under the plurality method, and rank the candidates under the extended plurality method.
- **e.** Find the winner under the Instant Runoff method, and rank the candidates under the extended Instant Runoff method.
- **f.** Find the winner under the Borda count method, and rank the candidates under the extended Borda count method.
- **g.** Find the winner under the pairwise comparisons method, and rank the candidates under the extended pairwise comparisons method.
- **h.** Of all the results you just obtained for this election under the four voting methods we have considered in the course, which ones, if any, violate the majority criterion? Which ones, if any, violate the Condorcet criterion? Explain briefly.
- **i.** Suppose we are using the Instant Runoff method, and for some reason, after holding the election and declaring a winner, we discover that B was actually ineligible to run, and therefore we hold a revote in which B is never a candidate at all. Does the result of the revote violate the monotonicity criterion, and why or why not? Does it violate the Independence of Irrelevant Alternatives criterion, and why or why not?

2. In any election, is a majority candidate automatically a Condorcet candidate, and if so, why? Is a Condorcet candidate automatically a majority candidate, and if so, why?

3. In each of the following scenarios, a violation of a fairness criterion of voting occurs, or merely appears to occur but actually does not, or we don't have enough information to tell.

- **a.** Ann has more first place votes than any other individual candidate, but Bob wins the election. Does this election result violate the majority criterion? Why or why not? Does this election result violate the Condorcet criterion? Why or why not?

b. Ann has more first place votes than all other candidates combined, but Bob wins the election. Does this election result violate the majority criterion? Why or why not? Does this election result violate the Condorcet criterion? Why or why not?

c. An election is held, and Bob is the winner. For some reason Ann is disqualified, and a revote is held. In the revote, Claire is the winner. Does this election result violate the monotonicity criterion? Why or why not? Does this election result violate the I.I.A. criterion? Why or why not?

d. An election is held, and Bob is the winner. For some reason Bob is disqualified, and a revote is held. In the revote, Claire is the winner. Does this election result violate the monotonicity criterion? Why or why not? Does this election result violate the I.I.A. criterion? Why or why not?

e. An election is held and Dave is the winner. For some reason a second election (a revote) is held, in which some voters change their ballots from what had been

1st choice	Claire		1st choice	Ann
2nd choice	Ann	in the first election, to	2nd choice	Dave
3rd choice	Dave		3rd choice	Bob
4th choice	Bob		4th choice	Claire

in the second election. When the votes are counted in the second election, Ann turns out to be the winner. Does this election result violate the monotonicity criterion? Why or why not? Does this election result violate the I.I.A. criterion? Why or why not?

f. Claire defeats every opponent in head to head competition, but Dave wins the election. Does this election result violate the majority criterion? Why or why not? Does this election result violate the Condorcet criterion? Why or why not?

g. Claire defeats Ann and Dave, and ties Bob in head to head competition; Claire never loses any head to head competition. Dave wins the election. Does this election result violate the majority criterion? Why or why not? Does this election result violate the Condorcet criterion? Why or why not?

4. In a certain election with 5 candidates, Zeke receives 51% of the first place votes and 49% of the last place votes. Of the four voting methods we have considered, under which voting method(s) is Zeke guaranteed to be the winner, and why specifically is he guaranteed to be the winner? Under which voting method(s) is it possible for him to lose? Explain your answers briefly.

5 If someone devises a voting method which never violates the Majority, Condorcet, or I.I.A. criteria, then what can you say about the voting method?

6 Is a majority candidate necessarily also a Condorcet candidate? Use the following example to think it through:

	11	6	4
1st choice	*C*	*B*	*A*
2nd choice	*A*	*A*	*B*
3rd choice	*B*	*C*	*C*

7. Suppose that Claire defeats Ann and Dave, and ties Bob in head to head competition; Claire never loses any head to head competition. Dave wins the election.

- Does this election result violate the Majority Criterion, or do we not have enough information to tell? Explain your reasoning.
- Does this election result violate the Condorcet Criterion, or do we not have enough information to tell? Explain your reasoning.

Chapter 2: The Mathematics of Power

1. Consider the weighted voting system [10: 9, 4, 3, 2].

a. Find the Banzhaf and Shapley-Shubik power distributions of [10: 9, 4, 3, 2].

$< P_1, P_2, P_3, P_4 >$ $< P_2, P_1, P_3, P_4 >$ $< P_3, P_1, P_2, P_4 >$ $< P_4, P_1, P_2, P_3 >$

$< P_1, P_2, P_4, P_3 >$ $< P_2, P_1, P_4, P_3 >$ $< P_3, P_1, P_4, P_2 >$ $< P_4, P_1, P_3, P_2 >$

$< P_1, P_3, P_2, P_4 >$ $< P_2, P_3, P_1, P_4 >$ $< P_3, P_2, P_1, P_4 >$ $< P_4, P_2, P_1, P_3 >$

$< P_1, P_3, P_4, P_2 >$ $< P_2, P_3, P_4, P_1 >$ $< P_3, P_2, P_4, P_1 >$ $< P_4, P_2, P_3, P_1 >$

$< P_1, P_4, P_2, P_3 >$ $< P_2, P_4, P_1, P_3 >$ $< P_3, P_4, P_1, P_2 >$ $< P_4, P_3, P_1, P_2 >$

$< P_1, P_4, P_3, P_2 >$ $< P_2, P_4, P_3, P_1 >$ $< P_3, P_4, P_2, P_1 >$ $< P_4, P_3, P_2, P_1 >$

b. Find all dictators, veto power players, and dummies (and explain how you can tell).

c. Which players (if any) can complain that their share of actual power is less than their share of total weight? Do Banzhaf and Shapley-Shubik give the same answer to this question?

2. In a weighted voting system with three players the winning coalitions are the following:

$\{P_1, P_2, P_3\}$, $\{P_1, P_2\}$, $\{P_1, P_3\}$

a. Find the Shapley-Shubik power distribution of the weighted voting system.

b. How many times more power does P_1 have than P_2?

3. In a weighted voting system with four players the winning coalitions are the following:

$\{P_1, P_2, P_3, P_4\}$, $\{P_1, P_2, P_3\}$, $\{P_1, P_2, P_4\}$, $\{P_1, P_3, P_4\}$, $\{P_1, P_2\}$, $\{P_1, P_3\}$, $\{P_1, P_4\}$

- **a.** Find the Banzhaf power distribution of this voting system.
- **b.** Find all dictators, veto power players, and dummies (and explain how you can tell).
- **c.** According to Banzhaf's approach to measuring power, how many times more power does a council member with veto power have than a council member without veto power?

4. Consider a simplified version of the UN Security Council in which there are six players (council members) P_1, P_2, P_3, P_4, P_5, and P_6. In order for a resolution to pass, a majority of council members must vote in favor of it. Moreover, P_1, P_2, P_3,and P_4 all have veto power while the others do not.

- **a.** Find the Banzhaf power distribution of this voting system.
- **b.** According to Banzhaf's approach to measuring power, how many times more power does a council member with veto power have than a council member without veto power?

5. Suppose that the weighted voting system [11: 9, 4, 4, 3] is implemented in the following situation:

A county consists of four districts, A, B, C, and D, whose populations are 90 thousand, 40 thousand, 40 thousand, and 30 thousand, respectively. Each district has one representative in the County Council, and casts a number of votes equal to the population (in tens of thousands) of his/her district; thus the district A representative casts 9 votes, the district B representative casts 4 votes, the district C representative casts 4 votes, and the district D representative casts 3 votes. 11 votes must be cast in favor of a motion in order to pass it.

- **a.** Find the Banzhaf power distribution of this weighted voting system.
- **b.** Which players, if any, are dictators, veto power players, or dummies?
- **c.** According to Banzhaf, which district or districts have a percentage of actual power which is less than their percentage of the total county population? Which district or districts have a percentage of actual power which is greater than their percentage of the total county population?
- **d.** If the quota could be changed, what value(s) of the quota would give all players veto power?

6. Suppose now that districts B and C merge into one "super-district" of 80,000 people. The players are now A with 9 votes, B-C with 8 votes (as a single bloc), and D with 3 votes.

- **a.** Find the Shapley-Shubik power distribution of the resulting weighted voting system, [11:9, 8, 3].
- **b.** According to Shapley and Shubik, which district or districts have a percentage of actual power which is less than their percentage of the total county population? Which district or districts have a percentage of actual power which is greater than their percentage of the

total county population? Keep in mind that B-C is now one district with 80,000 people, which casts 8 votes.

7. A university committee is made up of the president (P), a dean (D), two faculty members (F1 and F2), and two students (S1 and S2). In order for the committee to approve a proposal, the set of committee members voting in favor must be one of the following:
{P, D, F1, F2, S1, S2}, {P, D, F1, F2, S1}, {P, D, F1, F2, S2}, {P, D, F1, F2},
{P, D, F1, S1, S2}, and {P, D, F2, S1, S2}.

- **a.** Find the Banzhaf power distribution of this weighted voting system.
- **b.** How many times more powerful is the president than a student according to Banzhaf?
- **c.** Are there any dictators, veto power players, or dummies in this system?
- **d.** Find the pivotal player in each of the following sequential coalitions:

<P, D, F1, F2, S1, S2> < F2, S1, S2, P, D, F1 > < D, F1, F2, S1, S2, P>

Chapter 3: Mathematics of Apportionment

1. The town of Piscatawick has a police force of 40 officers, each of whom is assigned to one precinct. There are four police precincts: Downtown, East Side, North Side, and South Side. The number of officers apportioned to each precinct is based on the number of incidents last year in that precinct, given in the following table:

Precinct	# of incidents
Downtown	450
East Side	212
North Side	151
South Side	289

- **a.** Which are the seats, which are the states, and what plays the role of population in this apportionment problem? You are not asked here for numbers; instead answer in terms of officers, incidents, and precincts.
- **b.** Find the standard divisor for this apportionment problem.
- **c.** Explain in one or two sentences what the standard divisor represents in this problem (not how to calculate it); use the language of officers, incidents, and precincts to explain.
- **d.** Find the standard quota for each precinct.

2. This problem refers to the Piscatawick police department in problem 1 above.

- **a.** Find the Hamilton apportionment of officers to precincts based on number of incidents in the precinct (as presented in problem ExamReview1 above).
- **b.** Based on the number of officers each precinct receives in the Hamilton apportionment,

compute for each precinct the average number of incidents per officer. This need not be a whole number, since it is an average, and will be different for each precinct.

c. Which precinct has the most incidents per officer? Is this the precinct which is best served by police, or the worst?

3. This problem refers to the Piscatawick police department in problem 1 above.

a. Find the Jefferson apportionment of officers to precincts based on number of incidents in the precinct. [Hint: One of the following modified divisors will work: 26, 26.4, 26.8, 27.2, 27.6, or 28. Before trying any of these, first think whether the modified divisor should be greater than or less than the standard divisor.]

b. Based on the number of officers each precinct receives in the Jefferson apportionment, compute for each precinct the average number of incidents per officer. This need not be a whole number, since it is an average, and will be different for each precinct.

c. Which precinct has the most incidents per officer?

4. This problem refers to the Piscatawick police department in problem 1 above.

a. Find the Webster apportionment of officers to precincts based on number of incidents in the precinct (as presented in problem 1 above). [Hint: First, determine whether the standard divisor already works, and results in a Webster apportionment. If not, should the modified divisor be greater than the standard divisor or less? In any case, one of the following modified divisors will work: 27.3, 27.5, 27.7, or 27.9.]

b. Based on the number of officers each precinct receives in the Webster apportionment, compute for each precinct the average number of incidents per officer. This need not be a whole number, since it is an average, and will be different for each precinct.

c. Which precinct has the fewest incidents per officer?

5. This problem refers to the Piscatawick police department in problem 1 above.

Find the Huntington-Hill apportionment of officers to precincts based on number of incidents in the precinct. [Hint: First, determine whether the standard divisor already results in a Huntington-Hill apportionment. If not, should the modified divisor be greater than the standard divisor or less? In any case, one of the following modified divisors will work: 27.54, 27.55, 27.56, 27.57, 27.58, or 27.6.].

6. This problem pertains to the Alabama paradox.

a. Suppose that the Piscatawick police department described in problem ExamReview1 above hires one more officer, for a total of 41, but the number of incidents in each precinct remains the same. Find the Hamilton apportionment.

b. Does increasing the number of officers from 40 to 41 create an instance of the Alabama paradox? Explain how you can tell.

7. Based on a hypothetical apportionment given below, where 1000 seats are being apportioned:

State	Population	Standard Quota	Modified quota (when modified divisor equals 1030)	Apportionment under Jefferson's method
A	30000	27.91	29.13	29
B	6000	5.58	5.83	5
C	4500	4.19	4.37	4
D	2500	2.33	2.43	2

Do any violations of the Quota Rule occur in this apportionment? How can you tell?

Chapter 4: Mathematics of Fair Division

1. Mary and Neil share the rights to use a certain store location, but they have separate businesses, and only one can use the space at a time. To minimize the costs and hassles of turnover, they agree that each year, one should get the space from January 1 until a certain date, and the other should get the space from that date through the end of the year.

- Mary values each month from January through August twice as much as each month from September through December.
- Neil values each month from January through April three times as much as each month from May through December.

They agree to use the Divider-Chooser method to make a fair division of the access time to the store location. As always, we assume that neither one knows the other's preferences, i.e. each one makes choices based solely on his/her own system of values. They toss a coin, and the winner of the coin toss gets to be the Chooser.

Suppose that Neil loses the coin toss and is therefore the Divider.

a. At what point in the year does Neil make the division, and what are the resulting shares? Show the work which leads to your conclusion.

b. Which share does Mary choose, and how much is it worth to her as a fraction of what the whole year is worth to her? Show the work which leads to your conclusion.

c. Suppose that Mary and Neil get an expert to give each one a financial estimate of the value of having the store location for the whole year. For Mary, having the store location for the whole year is worth $240,000, while for Neil it is worth $300,000. If Neil is the Divider, what will his share be worth to him in dollars, and what will Mary's share be worth to her in dollars? Which

player receives a higher fraction of the total value of the year according to his/her own value system, and who receives a share worth more in dollars?

2. Eve, Fred, Gail, and Hank are four siblings who inherit a large piece of land. The land is not uniform, and has different features in different locations. A Divider is selected at random, and divides the land into 4 parts ("shares"), which as usual we will call s_1, s_2, s_3, and s_4.

a. Eve, Fred, Gail, and Hank each write their assessments of the value of each share, recorded in the following table. For each player, find his/her fair share in dollars (for which you'll need to find his/her total estimate of the value of the land), find his/her bid list, and record these in the table:

	Value of s_1	**Value of s_2**	**Value of s_3**	**Value of s_4**	**Minimum fair share amount in dollars**	**Bid list**
Eve	$150,000	$150,000	$150,000	$150,000	$	
Fred	$200,000	$100,000	$500,000	$100,000	$	
Gail	$200,000	$200,000	$200,000	$200,000	$	
Hank	$150,000	$100,000	$450,000	$200,000	$	

b. Who could have been the Divider, and how can you tell?

c. What is wrong with the following argument?

"Eve gets s_1 and Gail gets s_2. That leaves Fred and Hank, who both want s_3. Since Fred bid higher for it we give s_3 to Fred, and give s_4 to Hank (so he gets his second favorite share)"

d. There is a "standoff" here between Fred and Hank.

- What does it mean that there is a standoff?\
- Suppose that before the standoff is resolved, Eve is given s_1 and Gail is given s_2. Describe in words, as specifically as possible in light of the information available, what is done at this stage to resolve the standoff.

e. There is not enough information given to find the exact dollar value of the share which Fred will ultimately receive, after the standoff is resolved. Based on the available information, Fred will receive a share worth at least how much? Is this a fair share, in relation to the original problem of dividing the land among the four siblings, and how can you tell?

f. There is not enough information given to find the exact dollar value of the share which Hank will ultimately receive, after the standoff is resolved. Based on the available information, Hank will receive a share worth at least how much in dollars? Is this a fair share, in relation to the original problem of dividing the land among the four siblings, and how can you tell?

3. Ann, Bob, and Chris have equal claims to a store location, and are using the Lone Divider method to find a fair division of access to the location over the calendar year. We assume as usual that January 1 has to be one of the start/end dates when dividing the year into shares.

- Ann values each of the months February through July twice as much as each of other months (January, or August through December).
- Bob values each of the months January through March three times as much as each of the months April through December.

Chris values each of months of the year equally.

a. Suppose that Ann is the Divider. At what points does Ann divide the year? What are the shares which Ann produces, described in terms of which months (or parts of months) are contained in each share?
b. Which shares are fair shares for Bob? Which shares are fair shares for Chris?
c. Find a fair division of the year; indicate which share each player receives, and what it is worth to that player as a fraction of what the whole year is worth.
d. Can you find a different fair division of the year than the one you just found in part (c), or was that the only possible one? Indicate which share each player receives, and what it is worth to that player as a fraction of what the whole year is worth to him/her.

4. Bob, Dave, and Eve have equal claims to a store location, and are using the Lone Divider method to find a fair division of access to the location over the calendar year. We assume as usual that January 1 has to be one of the start/end dates when dividing the year into shares. Their relative preferences for the months of the year are recorded in the following table:

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Bob	2	2	2	1	1	1	1	1	1	1	1	1
Dave	0	0	0	1	1	1	1	2	2	2	2	2
Eve	1	1	2	1	1	1	2	3	3	3	3	3

a. Suppose that Bob is the Divider. At what points does Bob divide the year? What are the shares which Bob produces, described in terms of which months (or parts of months) are contained in each share?
b. Explain why a standoff occurs here, and what it means that there is a standoff.
c. Suppose that prior to resolving the standoff, Bob is given the first share in the calendar year. Describe a specific way in which the standoff can now be resolved; there is more than one way to do this, but you are asked to find just one!!! At the end, indicate which share each player receives, and how much that share is worth to the player as a fraction of what the whole year is worth to him/her.

5. Jason, Kelly, and Lauren use the Lone Chooser method to divide use of a retail space over the course of a year. Kelly is the Chooser, and Jason is the first Divider.

Each runs a different type of business. Their individual preferences are as follows:

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total
Jason	1	1	1	1	1	1	1	1	1	1	1	1	12
Kelly	1	1	1	1	1	1	1	1	1	0	0	0	9
Lauren	1	1	1	1	1	1	1	1	1	3	3	3	18

Division:

- **a.** Where does Jason make the first division?
- **b.** If we call the two shares into which Jason has divided the year s_1 and s_2, how much are each of s_1 and s_2 worth to Lauren, as a fraction (or percentage) of what the whole year is worth?
- **c.** Describe Lauren's share and Jason's share at the end of the division stage.

Subdivision:

- **d.** Describe the second division which Jason makes, i.e. describe (in terms of specific months or parts of months) the 3 subshares j_1, j_2, j_3 he creates from his share, and indicate how much each subshare is worth to him. You may describe the value to Jason in terms of "Jason points", or as a fraction of the value of the whole year.
- **e.** Describe the second division which Lauren makes, i.e. describe (in terms of specific months or parts of months) the 3 subshares l_1, l_2, l_3 she creates from her share, and indicate how much each subshare is worth to her. You may describe the value to Lauren in terms of "Lauren points", or as a fraction of the value of the whole year.

Selection: Now Kelly, the Chooser, returns.

- **f.** In the following table, write down how much each subshare of Jason's is worth to Kelly, and how much each subshare of Lauren's is worth to Kelly. You may describe the value to Kelly in terms of "Kelly points", or as a fraction of the value of the whole year.
- **g.** Which subshares does Kelly select?
- **h.** By filling out the following table, describe the final fair division of the year (indicating which months each player receives), and find the value of each player's final share as a fraction (or percentage) of the value of the entire year. Does each player receive a fair share, and how can you tell?

Player	Subshares received	Description (which months or parts of months)	Fraction of value of the entire year
Jason			
Kelly			
Lauren			

Chapter 5: Mathematics of Fair Distribution

1. Wendy and Xavier are siblings who have inherited a house from their parents. They are not willing to share the house (growing up together was enough), so one or the other will receive it when their parents' estate is settled. This is the only item which they have inherited, and they apply the Method of Sealed Bids to obtain a fair distribution. Wendy considers the house worth $400,000, but Xavier considers it worth $300,000.

a. Find the value of each player's minimum fair share, in dollars.
b. Carry out the allocation of the items (of which there is only one), and the first settlement.

First settlement:

	Items received (if any), and their value	$ initially paid to bank account	$ initially received from bank account
Wendy			
Xavier			

c. Find the surplus after the first settlement is over, and how much of the surplus each player receives.
d. Carry out the final settlement (for each player, indicate the items received and net amount of cash paid or received as of the end of the process).

Final settlement:

	Items received (if any), and their value	net $ paid to bank account after the surplus is distributed	net $ received from bank account after the surplus is distributed
Wendy			
Xavier			

2. Jerry, Elaine, George, and Kramer own some items jointly, and wish to find a fair distribution of them before they each move to a different part of the country. They use the Method of Sealed Bids to make the distribution of these 3 items, and write out their bids as follows:

	television	couch	Coffee maker
Jerry	$1000	$1800	$200
Elaine	$720	$1500	$180
George	$1200	$1100	$100
Kramer	$950	$1000	$150

a. Carry out the allocation of the items, and the first settlement.

First settlement:

	Items received (if any), and their value	$ initially paid to bank account	$ initially received from bank account
Jerry			
Elaine			
George			
Kramer			

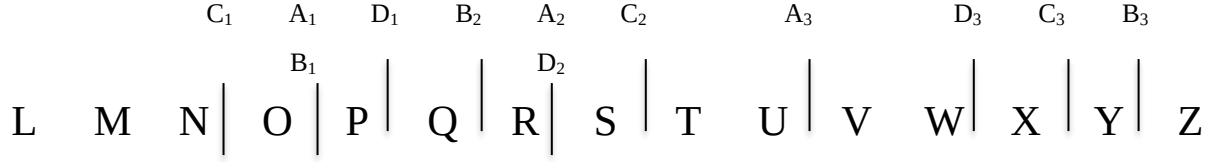
b. Find the surplus after the first settlement is over, and how much of the surplus each player receives.

c. Carry out the final settlement (for each player, indicate the items received and net amount of cash paid or received as of the end of the process).

Final settlement:

	Items received (if any), and their value	net $ paid to bank account after the surplus is distributed	net $ received from bank account after the surplus is distributed
Jerry			
Elaine			
George			
Kramer			

3. Ann (A), Bob (B), Claire (C), and Dave (D) use the Method of Markers to distribute a collection of 15 albums, labeled by letters L through Z. The arrangement of the albums, and the placement of the players' markers, is as follows:



Which items does each player receive? Which player receives his/her first minimum fair share, which receives his/her second minimum fair share, which receives his/her third minimum fair share, and which receives his/her fourth minimum fair share? Which items are leftovers?

4. Ed, Faith, and Greg must find a fair distribution of 3 oranges (O), 4 pears (P), and 5 apples (A), using the Method of Markers. Their individual preferences are as follows:

- Ed likes all types of fruit equally
- Faith likes only Apples and Pears, not Oranges
- Greg likes Oranges twice as much as Pears, and likes Pears and Apples equally

The items are arranged in the following order. Place each player's markers. [You don't need to find which share goes to which player; placing the markers is all you must do.]

A O O P A O A P A P A P

Chapter 6: The Mathematics of Growth and Finance

1. Consider the following formulas:

$$FV = PV(1 + i)^n \qquad FV = PMT\frac{(1+i)^n - 1}{i}$$

a. For each of the following formulas, under what circumstances is it the appropriate formula to use?
b. What does *n* mean in the first formula? What does *n* mean in the second formula?

2.

a. In a certain swamp, the mosquito population is increasing by 4 and three fourths percent each year. If it maintains that rate of growth, what will be the overall % increase after 30 years?
b. In another swamp sprayed aggressively with pesticides, the mosquito population is decreasing by 4 and three fourths percent each year. If it maintains that rate of decline, by what overall percentage will be population decline over the next 30 years?

3.

a. Suppose that on your 30th birthday (and only then), you put $3,000 into a savings account which pays 6% annual interest, compounded annually. How much will be in the account on your 50th birthday?
b. Now suppose that on your birthday *every* year starting when you turn 30, you put $3,000 into a savings account which pays 6% annual interest, compounded annually. How

much will be in the account on your 50th birthday? Assume that you do make a deposit on your 50th birthday, but it earns no interest.

c. In part b of this question, what is the total amount of interest you will have earned, as of your 50th birthday?

d. The total account value in part b is really the result of computing a long sum (even though you used a formula as a shortcut). Write out the first few terms and last few terms of that sum. How many terms are in the sum?

4. Find the APY of each of the following credit cards, and decide which one offers a better deal for the cardholder:

- Capital Two has an APR of 30% and compounds every 2 months.
- Lunchers Club Card has an APR of 29% and compounds twice per week.

Express the APYs you find in percent form, rounded to the nearest hundredth of a percent (i.e. to two decimal places *after* it is percent form, which requires that you have at least 4 decimal places of accuracy before converting to % form. To achieve this, avoid rounding until the very end of your computations).

5. Over a long period of time, the price of a certain house increased by 30%, but then decreased by 20%. What was the overall percentage change in the price of the house?

6. Suppose you expect that at the end of 2020 you will need to have $50,000 in savings. To achieve this goal you set up a savings plan, so that starting at the end of April 2015, and continuing at the end of every month thereafter until (and including) Dec. 31, 2020, you put the same amount of money into a savings account which pays 6% annual interest, compounded monthly. How much do you need to save each month? Assume that you do make a deposit on Dec. 31, 2020, but it earns no interest.

7. Suppose that you borrow $10,000 today at 7.28% annual interest compounded weekly.

a. How much will you owe 3 years from now?

b. What is the overall 3-year growth factor, by which your debt will grow?

8. Suppose you expect that on January 1, 2021 you will need to have $50,000 in savings. If you make one deposit on May 1, 2015 to a savings account which pays 6% annual interest, compounded monthly, and make no other deposits, how large does that deposit need to be in order to achieve your goal?

9. Explain the difference between APR and APY, and why it matters.

10. In this problem, we talk about the irritatingly unpredictable price of gasoline. Suppose on January, the price was $3.60 dollars per gallon. By February, the price had gone up 3.2%. By March, the price had gone down by 4.8% from the February price. By April, the price had gone up 5.8% from the March price. Finally, by May, the price had gone up by 2.4% from the April price.

a. What was the final price of gas by May? (Round your answer to the nearest cent.)
b. What percent change does this represent over the entire period?

11. Which would you rather invest your money into?

- Option #1: APR of 6.5% compounded annually
- Option #2: APR of 6.45% compounded monthly

Explain your answer.

12. You decide to deposit $200 at the end of every month into an IRA (retirement fund) with an APR of 4.8% compounded monthly. Let's say you plan to retire after 36 years.

a. How much money do you have in the account by the time you retire?
b. How much money is your very first $200 investment worth at the time you retire?

13. Suppose you wanted to save $5,000 after 5 years. You decide to start depositing money into a savings account earning 4% interest compounded monthly. How much should you deposit at the end of each month in order to reach your goal?

14. Sally decides to deposit $1,000 at the end of every three months (every quarter) into an IRA that earns 4.5% interest compounded quarterly. She works for another 40 years, then retires. Throughout the 40 years she continues to deposit $1,000 at the end of every three months and makes no withdrawals.

a. What is the future value of her **last** $1,000 investment at the time of her retirement?
b. How much money is in the account when she retires after 40 years?
c. What is the future value of her **second** $1,000 investment when she retires?

15. Jake can make payments of $350 each month. He has found a loan offer for 4 years at 3.3% compounded monthly.

a. How much can he afford to borrow now?
b. How much does he actually pay back for the loan?
c. How much Interest does he pay back for the loan?

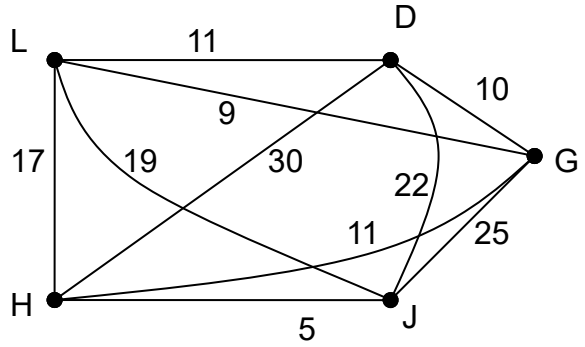
16. Suppose, instead of borrowing money now and repaying it, Jake started saving $350 at the end of each month. He invests it in an account earning 2.9% interest compounded monthly

(banks never offer as much interest as they charge). How much will he have in the account at the end of 4 years?

Chapter 7: Graph Theory

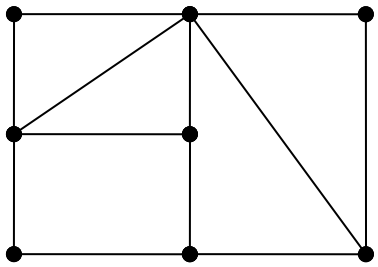
1. You have a supercomputer that can generate one million Hamilton circuits per second. Calculate in years how long it would take the supercomputer to generate all the Hamilton circuits in K_{18}. Round to the nearest year.

2. Maggie needs to go to the dry cleaners (D), the grocery store (G), Jiffy Lube (J), the library (L), and then return home (H). The travel times (in minutes) are given in the weighted graph shown below.



a. How many possible Hamilton circuits are there in this graph?
b. Using vertex H as the starting vertex, find the optimal tour (use the brute-force algorithm). What is the total travel time for this tour? (Hint: To save time, you cut the total amount of Hamiltonian circuits in half by not counting the same circuit but of reverse direction as distinct.)
c. Find the nearest-neighbor tour starting at vertex H. What is the total travel time for this tour?
d. Find the repetitive nearest-neighbor tour. What is the total travel time for this tour?
e. Rewrite the tour found in Part d so that it starts and ends at vertex H.
f. Find the cheapest-link tour. What is the total travel time for this tour?
g. Which method is guaranteed to find the tour with the least travel time?

3. Consider the network shown below



a. Find a spanning tree of the network.
b. Calculate the redundancy of the network

4. The chart below shows the distance between five cities (in kilometers).

	Florence	Milan	Naples	Rome	Venice
Florence	*	304	473	283	265
Milan	304	*	774	584	280
Naples	473	774	*	228	734
Rome	283	584	228	*	543
Venice	265	280	734	543	*

a. Use Kruskal's algorithm to find the MST connecting the six cities.
b. Find the weight of the MST