Math 115 Exam II ("Midterm II") Review

The second Math 115 full period exam covers Chapters 1.7-1.12 & Chapter 2 of the text. Material covered on the exam will be in relation to the syllabus for the course.

Group A that follows covers Chapter 1.8, providing study problems that will enable students to validate their algebra skills in relation to solving inequalities.

Subsequently, Groups B, C, etc. offers study problems related to all other chapters for this exam.

Students should be knowledgeable in solving problems related to the following functions: linear, quadratic, power, root, absolute value, and reciprocal among others. (See page 166 of the text.)

When preparing for the exam, students should not rely only on this review material. To best prepare students may want to review also their recitation quizzes, the applicable parts of the Chapter 1 & 2 practice tests from the text book, and the "Language of Functions" document posted on the Math department’s Math 115 course website.

Note also that answers to the problems that follow are at the end of the packet.

Group A: Algebra Skills

1. Solve each of the following for all EXACT real value(s) of x, expressing your answer in interval notation:

   a) \(-1 < 2 - (3 - x) \leq 5\)  
   b) \(2|\text{x} + 1| - 4 > 0\)  
   c) \(\text{x}^2 - \text{x} \leq 6\)  
   d) \(\frac{x - 3}{x + 1} \geq 0\)  
   e) \((\text{x}^2 - 4)(\text{x} + 1) > 0\)
Group B: Graphing

1. Sketch the graph and determine the x-intercept, y-intercept, local maximum value of the function, local minimum value of the function, and the interval over which the function is increasing for \( f(x) = x^3 - 6x - 1 \).

   \((\text{Accurate to one decimal place.})\)

2. Sketch the graph of the piece-wise defined function for each problem below. Be sure to label the end points of each piece.

   a) \( f(x) = \begin{cases} \sqrt{x-1} + 1 & \text{if } 1 < x < 5 \\ x - 1 & \text{if } x \geq 6 \end{cases} \)

   b) \( g(x) = \begin{cases} |x + 3| & \text{if } -5 < x < -2 \\ 1 - x^2 & \text{if } -1 \leq x < 2 \end{cases} \)

3. a) Using transformations sketch each of the following graphs:

   i) \( \sqrt{x - 5} + 2 \)  
   ii) \( -|x + 1| + 3 \)  
   iii) \( 2(x + 1)^2 - 1 \)

   b) Based on the graph of the function \( f \) below, on the same set of axes, graph

   i) \( g(x) = 2f(-x) - 1 \)  
   ii) \( g(x) = -f(x + 1) + 2 \)
Group C: Evaluating Functions, Domains & Inverses

Let $f(x) = x^2 - 2x - 8$, $g(x) = \sqrt{x-1}$ and $h(x)$ defined by the graph

1. Determine each of the following EXACTLY, expressing your answer in simplest form:
   
   a) $h(-2)$  
   b) $h^{-1}(-3)$  
   c) $(f - g)(2)$  
   d) $\left(\frac{h}{g}\right)(1)$  
   e) $(f \circ g)(5)$  
   f) $(f \circ g)(x+1)$

2. Determine the domain for each of the following, expressing your answer in interval notation.
   
   a) $f(x)$  
   b) $g(x)$  
   c) $h(x)$  
   d) $(f + g)(x)$

3. For $f(x)$ and $g(x)$, respectively, determine whether each is a 1-1 function. And, if so, find its inverse.

4. a) Determine the inverse function for each of the following:
   
   i) $f(x) = x^3 + 5$  
   ii) $f(x) = \frac{2x - 7}{x + 5}$

   b) For $f(x) = x^3 + 3x + 1$, find $f^{-1}(2)$, accurate to two decimal places.

   c) Based on the graph of the function $f$ below, on the same set of axis, graph $f^{-1}(x)$. 

3
Group D: Difference Quotients

Let \( f(x) = -x^2 + 2x \) and \( g(x) = \frac{3}{x - 3} \)

Determine each of the following, expressing your answer in simplest form:

1. \( \frac{f(x + h) - f(x)}{h}, \ h \neq 0 \)
2. \( \frac{g(x + h) - g(x)}{h}, \ h \neq 0 \)

Group E: Modeling (Applications)

1. At the end of 1997, the number of people living below the poverty level in the U.S. was 36.8 million, while at the end of 2002, the number was 31.8 million.
   a) Assuming that a linear relationship exists, write an equation expressing the number of millions of people, \( P \), living below the poverty level in relation to the number of years, \( x \), since 1997.
   b) Using the equation from part a), predict the number of people living below the poverty level at the end of 2003.
   c) What is the value of the slope, and what does it mean for this problem?

2. The maximum range of a projectile is directly proportional to the square of its velocity. A baseball pitcher throws a ball at 60 miles per hour, with a maximum range of 242 feet. What is the maximum range if he throws the ball ten miles per hour faster? (Accurate to the nearest foot.)
3. The width of a rectangular garden is 8 feet less than its length. If the garden is fenced in with 72 feet of wire fencing, what are the dimensions of the garden?

4. Mrs. B. invested $30,000; part at 5% simple annual interest and part at 8% simple annual interest. Her total interest earned for the year was $2,100. How much did she invest at each rate?

5. Two planes leave at 9 am from airports that are 2700 miles apart, and fly towards each other at speeds of 250 mph and 200 mph, respectively. At what time of day will the planes pass each other?

6. Tom and Jerry have to stuff and mail 1000 envelopes for a new marketing campaign. Jerry can do the job alone in 6 hours. However, if the two work together it will only take 4 hours to complete the job. How long would it have taken Tom to do the job by himself?

7. How many ounces of a 35% solution of sulfuric acid (and distilled water) must be mixed with 12 ounces of a 20% solution to get a 30% solution of sulfuric acid?

Group F: Miscellaneous

1. Given a circle whose equation is \((x + 3)^2 + y^2 = 52\).

   a) Determine the center and diameter of this circle, expressing the result in simplest form.

   b) Determine the **exact** value(s) of \(x\) for which the coordinate pair \((x,1)\) is on this circle.

2. a) Determine the equation of a line with \(x\)-intercept of -3 and \(y\)-intercept of -2.

   b) Then determine the equation of a perpendicular line with the same \(x\)-intercept. *(Express your answers in slope-intercept form.)*
Math 115 Exam II ("Midterm II") Review Answers

Group A: Algebra Skills

1. a) (0,6)  
   b) \((\infty, -3) \cup (1, \infty)\)  
   c) [-2,3]  
   d) \((\infty, -1) \cup [3, \infty)\)  
   e) \((-2, -1) \cup (2, \infty)\)

Group B: Graphing

1.

<table>
<thead>
<tr>
<th>x-intercept</th>
<th>(-2.4,0); (-0.2,0); (2.5,0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>y-intercept</td>
<td>(0,-1)</td>
</tr>
<tr>
<td>Local Maximum</td>
<td>4.7</td>
</tr>
<tr>
<td>Local Minimum</td>
<td>-6.7</td>
</tr>
<tr>
<td>Increasing</td>
<td>((\infty, 1.41) \cup (1.41, \infty))</td>
</tr>
</tbody>
</table>

2.

a)

b)
Group C: Evaluating Functions, Domains & Inverses

1. a) 3  b) 5  c) -9  d) Undefined  e) -8  f) $x - 2\sqrt{x} - 8$

2. a) $(-\infty, \infty)$  b) $[1, \infty)$  c) $(-\infty, 5]$  d) $[1, \infty)$

3. Only $g(x)$ is 1-1 function; $g^{-1}(x) = x^2 + 1, x \geq 0$

4. a i) $f^{-1}(x) = \sqrt{x - 5}$  ii) $f^{-1}(x) = \frac{5x + 7}{2 - x}$  b) 0.32

c) [Graph]

Group D: Difference Quotients

1. $-2x - h + 2$  2. $\frac{3}{(x + h - 3)(x - 3)}$

Group E: Modeling (Applications)

1. a) $P = -x + 36.8$  b) 30.8 million  c) Slope = -1 million people/year; Number of people living below the poverty level declines 1 million annually

2. 329 feet  3. Length = 22 feet; Width = 14 feet  4. $10,000 \@ 5\%; \$20,000 \@ 8\%$

5. At 3 pm  6. 12 hours  7. 24 ounces

Group F: Miscellaneous

1. a) Center = (-3,0), Diameter = $4\sqrt{13}$  b) $-3 \pm \sqrt{51}$

2. a) $y = \frac{2}{3}x - 2$  b) $y = \frac{3}{2}x + \frac{9}{2}$